EXERCISE 3-6

A

1. Express as a natural logarithm.

a. $3^x = 20$ **b.** $1.5^t = 8.2$ **c.** $e^t = 13$ **d.** $(0.72)^x = 9$

2. Evaluate each of the following using a calculator.

a. ln 50 **b.** ln 100 **c.** ln 2 **d.** ln 0.56

3. Use a calculator to solve each equation.

a. $\ln x = 0.045$ **b.** $\ln x = -0.15$ **c.** $e^x = 3$ **d.** $e^x = 0.56$

4. Evaluate where possible.

a. $\ln e^3$ **b.** $\ln e$ **c.** $\ln 1$ **d.** $\ln\left(\frac{1}{e^3}\right)$

e. $\ln 0$ **f.** $e^{\ln 2}$ **g.** $e^{\ln 5}$ **h.** $\ln e^{-2}$

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5. Express as a single logarithm.

a. $\ln 12 - \ln 4 + \ln 5$ **b.** $\frac{1}{2}\ln 9 - \ln 18$ **c.** $3\ln 2 - \frac{3}{2}\ln 16$

d. $\ln 5 + 3$ **e.** $\frac{1}{2}\ln 16 - 3$ **f.** $5 - \ln 2$

6. Solve each equation. Give each answer in terms of ln.

a. $e^x = 3$ **b.** $e^{2x} = 3$ **c.** $e^{x-2} = 5$ **d.** $e^{-x} = 3$

e. $e^{2-x} = 3$ **f.** $e^{5-x} = 2$ **g.** $e^{3-x} = e^{2x}$ **h.** $e^{2x-2} = e$

B

7. Express each formula as an exponential equation with base e.

a. $A = (1.56)^{2.1t}$ **b.** $V = V_0(1.6)^{-0.02t}$ **c.** $T = 20 + 80(2)^{-0.15t}$

8. Change each equation to an equivalent exponential equation with base e.

a. $y = 50(2^x)$ **b.** $y = (1.05)^x$ **c.** $y = 780\left(\frac{1}{2}\right)^x$

9. The population of Canada can be estimated using the formula $N = 18.2e^{0.0145(t-1961)}$, where 18.2 million was the population in 1961 and N is the number of people, in millions, in year t. In what year is the population of Canada expected to be 30 million?

10. The intensity, I_0, of light from a car's headlights is reduced to I after passing through d metres of fog according to the formula $I = I_0e^{-0.14d}$.
For what distance will the intensity be reduced to 0.01 of its original value?

11. The atmospheric pressure, P, at height h kilometres is given by the formula

$P = P_0e^{-kh}$.

The pressure at sea levels, P_0, is 101.3 kPa.

a. If $P = 89$ kPa when $h = 1$, find the value of k.
b. Calculate the pressure at a height of 2 km.

12. At 100°C, hot water cools in a room with temperature 20° C according to the formula

$T = 20 + 80e^{-0.03t}$, where T is the temperature t minutes after cooling begins.

How long will it take for the water to cool to 40° C in the room?

13. The speed, V, in kilometres per hour, of a boat travelling through water, t seconds after the engine is shut off, is given by

$V = V_0e^{-0.02t}$, where V_0 is the speed when the engine is shut off.

If a boat is moving at 8 km/h when the engine is shut off, how long will it take for the boat to reduce speed to 2 km/h? to 0.5 km/h?

14. When sunlight passes through water, it loses intensity as it penetrates to greater depths according to the formula

$I = I_0e^{-kd}$, where I_0 is the intensity of light at the surface of the water and I is the intensity, d metres below the surface.
Suppose that $I = 0.3012\,I_0$ at a depth of 100 m.

a. Find the value of k.
b. Find the depth at which the intensity is decreased to 1% of the surface intensity.

C **15.** When gamma-rays are passed through a heavy material, the rays are absorbed. The intensity, I, of the rays after absorption is given by the equation

$I = I_0e^{-kd}$, where I_0 is the intensity of the rays without a barrier, d is the thickness of the material (in centimetres), and k is a constant, determined by the material used.

If gamma rays are sent through a heavy material that is 7 cm thick, the intensity is reduced to $\frac{1}{8}$, or 0.125, of its original value.

a. Find the value of k, correct to four decimal places.
b. What thickness of heavy material is needed to reduce the intensity of the rays to 2% of the original intensity?

16. Solve.

a. $e^{2x} - e^x - 6 = 0$ **b.** $e^{4x} - 13e^{2x} + 36 = 0$