REVISION SET C - PAPER 1 & PAPER 2 STYLE QUESTIONS

1. (a) If $\overrightarrow{OA} = 3i - 2j + k$ and $\overrightarrow{OB} = i + 2k$, find $2\overrightarrow{OB} + 3\overrightarrow{AO}$.

(b) The angle between the vectors $u = 2i - j + 3k$ and $v = i + 4j - 2k$ is θ.

Find a given that $\cos\theta = \dfrac{a}{\sqrt{14 \times 21}}$.

(c) Find a unit vector perpendicular to $2i + j - 3k$.

2. Find, in i. parametric form ii. Cartesian form
the line of intersection of the planes $3x + 2y - z = 6$ and $x + 4y - z = -1$.

3. Find the cosine of the acute angle between the lines
$x = -1 + 2\lambda, y = 1 + 3\lambda, z = 2 - \lambda$ and $x = 7 + 5\mu, y = -8 - 3\mu, z = -2 + \mu$.

4. If $A = \begin{bmatrix} 2 & -1 \\ -1 & 2 \end{bmatrix}$, $X = \begin{bmatrix} x \\ y \end{bmatrix}$ where $x, y \neq 0$ and $AX = kX$ where k is a constant, find the values of k for which the equation is true.

5. The position vector of two particles, A and B is given by $r_A = ati + (by - 5t^2)j + t^2k$ and $r_B = 8ti - 4tj + 4k$. If the two particles collide at a point in space find when and where they collide and the values of a and b.

6. Find A^2 if $A = \begin{bmatrix} 0 & 2 & 1 \\ 1 & 1 & 1 \\ -1 & -2 & -2 \end{bmatrix}$. **Hence** solve the system equations $\begin{aligned} x + 2y + 2z &= 1 \\ 2y + z &= -2 \\ x + y + z &= 0 \end{aligned}$.

7. The matrices A, B and X are such that $A^{-1} = \frac{1}{2}A$, $B^{-1} = \frac{1}{9}B$ and $ABX = I$, where I is the unit matrix, find BX, X and AX in terms of A and B.

(b) Find the time taken, to the nearest minute, for the salt concentration to rise to 6 grams per litre.

8. Let the position vectors of points A and B be $\overrightarrow{OA} = -2i - j + k$ and $\overrightarrow{OB} = i - 2j - k$.

Find (a) $\overrightarrow{AB}$.

(b) the angle AOB to the nearest degree.

(c) the position vector of a point C if B is the mid-point of AC.

9. Let $A = \begin{bmatrix} 1 & 2i \\ -2i & 1 \end{bmatrix}$ where $i^2 = -1$.

(a) Show that $A^2 = 2A + 3I$.

(b) Find A^{-1} in terms of A and I.

10. The position vectors of particles A, B and C from a fixed point O, at any time t, are given by $\boldsymbol{r}_A = 2t^2\boldsymbol{i} + (t+1)\boldsymbol{j}$, $\boldsymbol{r}_B = 2t\boldsymbol{i} + (2t-4)\boldsymbol{j}$ and $\boldsymbol{r}_C = bt^3\boldsymbol{i} + (bt+4)\boldsymbol{j}$ where b is a constant.

(a) Find the minimum distance from O to the path of particle B.

(b) If particles A and C collide, find when they collide and the value of b.

11. If $\begin{bmatrix} a & 2 \\ 1 & 1 \end{bmatrix}\begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 1 \\ 2 \end{bmatrix}$, where $a \in \mathbb{R}$

(a) find $\begin{bmatrix} x \\ y \end{bmatrix}$ as a matrix in terms of a.

(b) state when $\begin{bmatrix} x \\ y \end{bmatrix}$ is not defined.

(c) find x and y if $a = 3$.

12. (a) The triangle ABC is such that $\boldsymbol{AB} = 3\boldsymbol{i} + 6\boldsymbol{j} - 2\boldsymbol{k}$ and $\boldsymbol{AC} = 4\boldsymbol{i} - \boldsymbol{j} + 3\boldsymbol{k}$.

Find i. $\angle BAC$ ii. the area of ΔABC

(b) PQRS is a trapezium with $\overrightarrow{PQ} = \boldsymbol{p}$, $\overrightarrow{PS} = \boldsymbol{s}$ and $\overrightarrow{SR} = 3\boldsymbol{p}$. T is the midpoint of [QR]. Express the following in terms of $\boldsymbol{p}$ and $\boldsymbol{s}$.

i. $\overrightarrow{PR}$ ii. $\overrightarrow{QR}$ iii. $\overrightarrow{PT}$ iv. $\overrightarrow{ST}$

13. (a) Given that $M = \begin{bmatrix} \cos\theta & -\sin\theta \\ \sin\theta & \cos\theta \end{bmatrix}$, find $det(M)$.

(b) If $A = \begin{bmatrix} 2 & a \\ 0 & 1 \end{bmatrix}$, deduce an expression for A^n, $n \geq 1$, .

(c) Find the value of a such that the matrix $M = \begin{bmatrix} 3 & 2 & 1 \\ 0 & 1 & 3 \\ -1 & a & 1 \end{bmatrix}$ is singular.

14. The position vectors of the points A, B and C are $\boldsymbol{i} - \boldsymbol{j} + 2\boldsymbol{k}$, $2\boldsymbol{i} + \boldsymbol{j} + 4\boldsymbol{k}$ and $3\boldsymbol{i} + 4\boldsymbol{k}$ respectively. Find

(a) the angle BAC to the nearest degree.

(b) the area of the triangle ABC.

(c) a vector equation of the plane ABC.

15. (a) Find the i. vector ii. parametric and iii. Cartesian equation of the line through the point with position vector $\begin{pmatrix} 2 \\ -3 \end{pmatrix}$ & parallel to $\begin{pmatrix} 3 \\ 7 \end{pmatrix}$.

(b) Find the position vector of the point of intersection of the lines

$$l_1 : \boldsymbol{r} = \begin{pmatrix} 14 \\ -1 \end{pmatrix} + \lambda\begin{pmatrix} 5 \\ -4 \end{pmatrix} \text{ and } l_2 : \boldsymbol{r} = \begin{pmatrix} 9 \\ -4 \end{pmatrix} + \mu\begin{pmatrix} -4 \\ 6 \end{pmatrix}$$

(c) i. Are the straight lines $x - 1 = \frac{y}{3} = \frac{z-1}{4}$ and $\frac{x-2}{4} = 3 - y = z$ parallel?

ii. Find the point(s) of intersection of the lines in i. What do you conclude?

16. Find the acute angle between the lines $r = \begin{pmatrix}1\\2\\4\end{pmatrix} + \lambda\begin{pmatrix}4\\3\\1\end{pmatrix}$ and $r = \begin{pmatrix}-1\\-4\\1\end{pmatrix} + \mu\begin{pmatrix}3\\2\\3\end{pmatrix}$.

17. The position vectors of the points A, B and C are given by $\overrightarrow{OA} = \boldsymbol{i} + 2\boldsymbol{j} + 2\boldsymbol{k}$, $\overrightarrow{OB} = \boldsymbol{i} + a\boldsymbol{j} - 2\boldsymbol{k}$ and $\overrightarrow{OC} = b\boldsymbol{i} + 3\boldsymbol{j} + c\boldsymbol{k}$ where a, b and c are constants. Find

(a) a if $\overrightarrow{OA}$ is perpendicular to $\overrightarrow{OB}$.

(b) b and c if O, A and C are collinear.

18. Find a unit vector perpendicular to both $2\boldsymbol{i} + \boldsymbol{j} + \boldsymbol{k}$ and $3\boldsymbol{j} - 2\boldsymbol{k}$.

19. Let $A = \begin{bmatrix}-2 & -3 & -1\\1 & 2 & 1\\3 & 3 & 2\end{bmatrix}$. Find all real numbers k such that the matrix $A - kI$ has no inverse.

20. Let $A = \begin{bmatrix}1 & 1 & -1\\3 & 4 & 6\end{bmatrix}$, $B = \begin{bmatrix}2 & 0\\-2 & 1\end{bmatrix}$, $C = \begin{bmatrix}1\\7\end{bmatrix}$ and $X = \begin{bmatrix}x\\y\\z\end{bmatrix}$.

(a) Evaluate i. BC ii. B^{-1}

(b) Find the equations of the planes produced from the matrix equation $AX = C$.

(c) Give the vector equation of the straight line where the two planes in (b) meet.

21. OABC, DEFG is a cuboid with $\overrightarrow{OA} = x\boldsymbol{i}, x > 0$, $\overrightarrow{OC} = 3\boldsymbol{j}$ and $\overrightarrow{OD} = 4\boldsymbol{k}$.

(a) Find x if the angle between the diagonals given by $\overrightarrow{OF}$ and $\overrightarrow{AG}$ is a right angle.

(b) Also find x if the same diagonals are 60° to each other.

22. State the position vectors, **OA** and **OB** of the points with coordinates A(2, –2, 1) and B(4, 0, –3) respectively. Find the angle between the vectors **OA** and **OB**.

23. If $A = \begin{bmatrix}1 & 0 & 2\\3 & 1 & 1\\-1 & 0 & -1\end{bmatrix}$ and $B = \begin{bmatrix}-1 & 0 & -2\\2 & 1 & 5\\1 & 0 & 1\end{bmatrix}$, evaluate AB and use your result to solve the simultaneous equations

$$\begin{aligned} x + 2z &= 1\\ 2x + y + 5z &= 1\\ x + z &= 2 \end{aligned}$$

24. Suppose $A = \begin{bmatrix}1 & 2\end{bmatrix}$, $B = \begin{bmatrix}3\\4\end{bmatrix}$, $C = \begin{bmatrix}1 & 2\\2 & 4\end{bmatrix}$, $D = \begin{bmatrix}1 & 2\\3 & 4\end{bmatrix}$. For each of the following, either find the matrix or give a reason why it cannot be found

(a) AB (b) BA (c) $C + D$ (d) $C^{-1}D$

25. Are the points P(3, 1, 0), Q(2, 2, 2) and R(0, 4, 6) collinear?

26. (a) Find the direction ratios and direction cosines of the lines

i. $l_1 : x = 2+\lambda,\ y = 2-\lambda,\ z = 2\lambda$

ii. $l_2 : x = 2+3t,\ y = 3+6t,\ z = 4+2t$

(b) Find the coordinates of the point of intersection of the lines l_1 and l_2.

27. Find the equation of the line throught A(1, 2, –3) and parallel to the line defined by the vector equation, $\boldsymbol{r} = \begin{pmatrix} 0 \\ 1 \\ -2 \end{pmatrix} + \lambda\begin{pmatrix} 3 \\ 2 \\ 1 \end{pmatrix}$

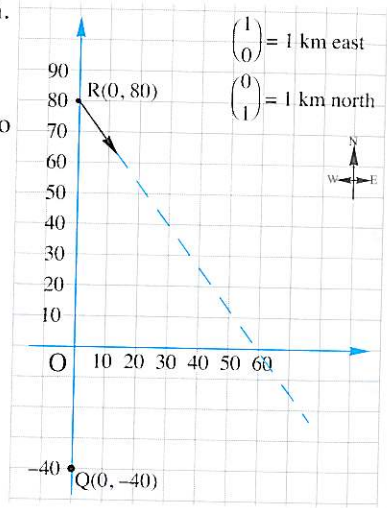
28. Find the point of intersection of the lines $r = \begin{pmatrix} 3 \\ 2 \end{pmatrix} + \lambda\begin{pmatrix} -1 \\ 1 \end{pmatrix}$ and $r = \begin{pmatrix} -2 \\ 1 \end{pmatrix} + \mu\begin{pmatrix} 2 \\ 3 \end{pmatrix}$

29. If the vectors $\begin{pmatrix} 3x \\ x+1 \end{pmatrix}$ and $\begin{pmatrix} x-4 \\ 4 \end{pmatrix}$ are perpendicular show that $3x^2 - 8x + 4 = 0$.

Hence find those values of x for which the vectors are perpendicular.

30. Particle A heads off on the path shown in the diagram.

Its velocity is given by the vector $\begin{pmatrix} 3 \\ -2 \end{pmatrix}$ ms^{-1}.



(a) Find the position vector of particle A relative to the origin O, t hours after its departure from point R(0, 80).

(b) Find particle A's position 2 hours after departure.

A second particle, B, is at rest at the point Q(0, –40).

(c) If particle B moves off with a velocity of $\begin{pmatrix} 4 \\ 1 \end{pmatrix}$ ms^{-1} at the same time that A departs, will the two particles collide? If not, when should particle B start to move so that the two particles will collide?

31. The ship, Excalibur, positioned 4 km east and 2 km north of a harbour sets off at a constant velocity of $\begin{pmatrix} 2 \\ 3 \end{pmatrix}$ kmh^{-1}. Fog has rendered visibility to a minimum.

(a) Find the position vector of Excalibur relative to the harbour, t hours after it sets off.

A lighthouse, no longer in operation, is located 25 km east and 13 km north of the harbour

(b) Find the position vector of Excalibur **relative to the lighthouse**.

(c) Hence, determine the closest that Excalibur will get to the lighthouse.