

EXERCISES 17.2

1. If Z is a standard normal random variable, find

(a) $p(Z > 2)$	(b) $p(Z < 1.5)$	(c) $p(Z \geq 0.5)$
(d) $p(Z \leq 1.2)$	(e) $p(Z \geq 1.5)$	(f) $p(Z \leq 2)$

2. If Z is a standard normal random variable, find

(a) $p(Z > -2)$	(b) $p(Z < -1.5)$	(c) $p(Z \geq -0.5)$
(d) $p(Z \leq -1.2)$	(e) $p(Z \geq -1.5)$	(f) $p(Z \leq -2)$

3. If Z is a standard normal random variable, find

(a) $p(0 \leq Z \leq 1)$	(b) $p(1 \leq Z \leq 2)$	(c) $p(1.5 \leq Z < 2.1)$
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4. If Z is a standard normal random variable, find

(a) $p(-1 \leq Z \leq 1)$	(b) $p(-2 \leq Z \leq -1)$	(c) $p(-1.5 \leq Z < -0.1)$
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5. If X is a normal random variable with mean $\mu = 8$ and variance $\sigma^2 = 4$. Find:

(a) $p(X \geq 6)$	(b) $p(5 < X \leq 8)$	(c) $p(X < 9.5)$
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6. If X is a normal random variable with mean $\mu = 100$ and variance $\sigma^2 = 25$. Find:

(a) $p(X \geq 106)$	(b) $p(105 < X \leq 108)$	(c) $p(X < 95)$
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7. If X is a normal random variable with mean $\mu = 60$ and standard deviation $\sigma = 5$. Find:

(a) $p(X \geq 65)$	(b) $p(55 < X \leq 65)$	(c) $p(50 \leq X < 55)$
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8. Scores on a test are normally distributed with a mean of 68 and a standard deviation of 8. Find the probability that a student scored:
 - (a) at least 75 on the test
 - (b) at least 75 on the test given that the student scored at least 70.
 - (c) In a group of 50 students, how many students would you expect to score between 65 and 72 on the test.

9. If X is a normally distributed variable with a mean of 24 and standard deviation of 2, find:

(a) $p(X > 28 X \geq 26)$	(b) $p(26 < X < 28 X \geq 27)$
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10. The heights of men are normally distributed with a mean of 174 cm and a standard deviation of 6 cm. Find the probability that a man selected at random:

(a) is at least 170 cm tall	(b) is no taller than 180 cm
(c) is at least 178 cm given that he is at least 174 cm.	

11. If X is a normal random variable with a mean of 8 and a standard deviation of 1, find the value of c , such that

(a) $p(X > c) = 0.90$	(b) $p(X \leq c) = 0.60$
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12. If X is a normal random variable with a mean of 50 and a standard deviation of 5, find the value of c , such that

(a) $p(X \leq c) = 0.95$	(b) $p(X \geq c) = 0.95$	(c) $p(-c \leq X \leq c) = 0.95$
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13. The Board of Examiners have decided that 80% of all candidates sitting the Mathematical Methods Exam will obtain a pass grade. The actual examination marks are found to be normally distributed with a mean of 45 and a standard deviation of 7. What is the lowest score a student can get on the exam to be awarded a pass grade?
14. The weight of a population of women is found to be normally distributed with mean 62.5 kg. If 15% of the women weigh at least 72 kg, find the standard deviation of their weight.
15. The weights of a sample of a species of small fish are normally distributed with a mean of 37grams and a standard deviation of 3.8grams. Find the percentage of fish that weigh between 34.73 and 38.93grams. Give your answer to the nearest whole number.
16. The weights of the bars of chocolate produced by a machine are normally distributed with a mean of 232grams and a standard deviation of 3.6grams. Find the proportion of the bars that could be expected to weigh less than 233.91grams.
17. For a normal variable, X , $\mu = 196$ and $\sigma = 4.2$. Find:
(i) $p(X < 193.68)$ (ii) $p(X > 196.44)$ (iii) $p(193.68 < X < 196.44)$
18. The circumferences of a sample of drive belts produced by a machine are normally distributed with a mean of 292cm and a standard deviation of 3.3cm. Find the percentage of the belts that have diameters between 291.69cm and 293.67cm.
19. A normally distributed variable, X , has a mean of 52. $p(X < 51.15) = 0.0446$. Find the standard deviation of X .
20. The lengths of the drive rods produced by a small engineering company are normally distributed with a mean of 118cm and a standard deviation of 0.3cm. Rods that have a length of more than 118.37cm are rejected. Find the percentage of the rods that are rejected. Give your answer to the nearest whole number.
21. After their manufacture, the engines produced for a make of lawn mower are filled with oil by a machine that delivers an average of 270ml of oil with a standard deviation of 0.7ml. Assuming that the amounts of oil delivered are normally distributed, find the percentage of the engines that receive more than 271.12ml of oil. Give your answer to the nearest whole number.
22. A sample of detergent boxes have a mean contents of 234grams with a standard deviation of 4.6grams. Find the percentage of the boxes that could be expected to contain between 232.22 and 233.87grams. Give your answer to the nearest whole number.
23. A normally distributed variable, X , has a mean of 259. $p(X < 261.51) = 0.9184$. Find the standard deviation of X .
24. A normally distributed variable, X , has a standard deviation of 3.9. Also, 71.37% of the data are larger than 249.8. Find the mean of X .
25. The times taken by Maisie on her way to work are normally distributed with a mean of 26 minutes and a standard deviation of 2.3 minutes. Find the proportion of the days on which Maisie's trip takes longer than 28 minutes and 22 seconds.

26. In an experiment to determine the value of a physical constant, 100 measurements of the constant were made. The mean of these results was 138 and the standard deviation was 0.1. What is the probability that a final measurement of the constant will lie in the range 138.03 to 139.05?
27. In an experiment to determine the times that production workers take to assemble an electronic testing unit, the times had a mean of 322 minutes and a standard deviation of 2.6 minutes. Find the proportion of units that will take longer than 324 minutes to assemble. Answer to 2 significant figures.
28. A normally distributed variable, X , has a standard deviation of 2.6. $p(X < 322.68) = 0.6032$. Find the mean of X .
29. The errors in an experiment to determine the temperature at which a chemical catalyst is at its most effective, were normally distributed with a mean of 274°C and a standard deviation of 1.2°C . If the experiment is repeated what is the probability that the result will be between 275°C and 276°C ?
30. The weights of ball bearings produced by an engineering process have a mean of 215g with a standard deviation of 0.1g. Any bearing with a weight of 215.32g or more is rejected. The bearings are shipped in crates of 10000. Find the number of bearings that might be expected to be rejected per crate.
31. If $X \sim N(\mu, 12.96)$ and $p(85.30 < X < 89.01) = 0.3933$. Find the mean of X .
32. At a Junior track and field meet it is found that the times taken for children aged 14 to sprint the 100 metres race are normally distributed with a mean of 15.6 seconds and standard deviation of 0.24 seconds. Find the probability that the time taken for a 14 year old at the meet to sprint the 100 metres is
- less than 15 seconds
 - at least 16 seconds
 - between 15 and 16 seconds.
 - On one of the qualifying events, 8 children are racing. What is the probability that 6 of them will take between 15 and 16 seconds to sprint the 100 metres?
33. Rods are manufactured to measure 8 cm. Experience shows that these rods are normally distributed with a mean length of 8.02 cm and a standard deviation of 0.04 cm. Each rod costs \$5.00 to make and is sold immediately if its length lies between 8.00 cm and 8.04 cm. If its length exceeds 8.04 cm it costs an extra \$1.50 to reduce its length to 8.02 cm. If its length is less than 8.00 cm it is sold as scrap metal for \$1.00.
- What is the average cost per rod?
 - What is the average cost per usable rod?
34. The resistance of heating elements produced are normally distributed with mean 50 ohms and standard deviation 4 ohms.
- Find the probability that a randomly selected element has resistance less than 40 ohms.
 - If specifications require that acceptable elements have a resistance between 45 and 55 ohms, find the probability that a randomly selected element satisfies these specifications.

- ii. A batch containing 10 such elements is tested. What is the probability that exactly 5 of the elements satisfy the specifications?
- (c) The profit on an acceptable element, i.e., one that satisfies the specifications, is \$2.00, while unacceptable elements result in a loss of \$0.50 per element. If \$ P is the profit on a randomly selected element, find the profit made after producing 1000 elements.
- 35.** (a) Find the mean and standard deviation of the normal random variable X , given that $P(X < 50) = 0.05$ and $P(X > 80) = 0.1$.
- (b) Electrical components are mass-produced and have a measure of ‘durability’ that is normally distributed with mean μ and standard deviation 0.5. The value of μ can be adjusted at the control room. If the measure of durability of an item scores less than 5, it is classified as defective. Revenue from sales of non-defective items is \$ S per item, while revenue from defective items is set at \$ $\frac{1}{10}S$. Production cost for these components is set at \$ $\frac{1}{10}\mu S$. What is the expected profit per item when μ is set at 6?
- 36.** From one hundred first year students sitting the end-of-year Botanical Studies 101 exam, 46 of them passed while 9 were awarded a high distinction.
- (a) Assuming that the students’ scores were normally distributed, determine the mean and variance on this exam if the pass mark was 40 and the minimum score for a high distinction was 75.
- Some of the students who failed this exam were allowed to sit a ‘make-up’ exam in early January of the following year. Of those who failed, the top 50% were allowed to sit the ‘make-up’ exam.
- (b) What is the lowest possible score that a student can be awarded in order to qualify for the ‘make-up’ exam?
- 37.** The heights of men in a particular country are found to be normally distributed with mean 178 cm and a standard deviation of 5 cm. A man is selected at random from this population.
- (a) Find the probability that this person is
- at least 180 cm tall.
 - between 177 cm and 180 cm tall.
- (b) Given that the person is at least 180 cm, find the probability that he is
- at least 184 cm.
 - no taller than 182 cm.
- (c) If ten such men are randomly selected, what are the chances that at least two of them are at least 176 cm?

IB Math SL1 Normal Distribution Problems Worksheet

Problems:

1. A normal distribution has a mean of 20 and a standard deviation of 4.
 - a) Find the percentile rank for a data value of 17 (i.e. find the percentage of data values that are below the data value of 17.
 - b) Find the data value such that approximately 70% of the data values are below it. (That is find the data value corresponding to the 70th percentile).
 - c) Find the data value such that approximately 95% values are above it
 - d) Find the data value for which 95% of the scores are below it.
 - e) Find the range of data values corresponding to the middle 50% of the distribution of scores.

2. An oil exploration company takes X-wave readings to determine where to drill for oil. The company's geologists have determined that X-wave readings taken above oil reservoirs will be normally distributed with a mean of 8.3 and a standard deviation of 2.1.
 - a) The company will drill for oil anywhere it finds an X-wave reading greater than x , with x to be determined. How should x be chosen to ensure that there is a 95% probability that the company will drill when it takes a reading above an oil reservoir? (That is so it will drill 95% of the oil reserves).
 - b) When X-wave readings are taken above ground where there are no oil reserves, the readings are normally distributed with a mean of 5.2 and a standard deviation of 1.3. If your answer to a) is adopted, what is the probability that a reading taken at a site where there is no oil will result in a decision to drill

3. The ages of subscribers to the Monon Daily newspaper are normally distributed with a mean of 35.5 years and a standard deviation of 4.8. (Source: *Fundamentals of Probability, Ghahramani*)
 - a) What is the percentage of subscribers that are more than 35.5 years?
 - b) What is the percentage of subscribers between 30 and 40 years?

4. Adult women are normally distributed with a mean of 63.6 and a standard deviation of 2.5 inches. Find the standard score (z-score) and percentile for each of the following heights:
 - a) 65 inches
 - b) 63 inches
 - c) What percentage are between 63 and 65 inches.