

Chapter 3 Test – Logarithmic and Exponential Functions

NO CALC

PART A

1. Evaluate the following expressions:

a.  $\ln \sqrt{e} - \ln e^3 + \ln \frac{1}{\sqrt[3]{e}}$

- i. (a)  $-\frac{1}{18}$
- ii. (b)  $-\frac{13}{6}$
- iii. (c)  $\frac{1}{2}$
- iv. (d)  $-\frac{17}{6}$
- v. (e) none of the above (enter alternative answer:           )

b.  $(4^{\log_2 8}) \times (4^{\log_4 \sqrt{32}})$

- i. (a)  $64\sqrt{2}$
- ii. (b)  $32\sqrt{2}$
- iii. (c)  $256\sqrt{2}$
- iv. (d)  $4^{\frac{11}{2}}$
- v. (e)  $4^{\frac{15}{2}}$

c.  $\log_3 81 + \log_2 \sqrt[3]{16} - \log_8 \sqrt{2}$

- i. (a)  $5\frac{1}{6}$
- ii. (b)  $16\frac{5}{6}$
- iii. (c)  $3\frac{5}{6}$
- iv. (d)  $\log_6 \left( \frac{81 \cdot \sqrt[3]{16}}{\sqrt{2}} \right)$  which cannot be simplified without a calculator
- v. (e) none of the above (enter alternative answer:           )

2. Write T or F (true or false) for the following statements:

(a)	$\ln(x+2) = \ln(x) + \ln(2)$	(f)	$\log_3(7x) = 7 \log_3 x$
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(b)	$\log(5x) = \log(5) + \log(x)$	(g)	$\ln \frac{x}{5} = \ln x - \ln 5$
(c)	$\log \frac{x}{4} = \frac{\log x}{\log 4}$	(h)	$\log_4 x^3 = 3 \log_4 x$
(d)	$\log_5 x^2 = (\log_5 x)(\log_5 x)$	(i)	$e^{\ln x + \ln 6} = 6x$
(e)	$2^{x+3} = 8 \cdot 2^x$	(j)	$a^{\log_a x} = b^{\log_b x}$

3. Simplify  $\ln(e^{4e})^{-1}$ .

a. (a)  $\frac{1}{\ln e^{4e}}$

b. (b)  $\frac{1}{4e}$

c. (c)  $-4e$

d. (d)  $-4$

e. (e)  $\frac{1}{4}$

4. If  $\log_5(cd - k) = u$ , then find an expression for  $d$ .

a. (a)  $d = \frac{k + \frac{u}{\log 5}}{c}$

b. (b)  $d = \frac{1}{c}(5^u + k)$

c. (c)  $d = \frac{u^5 + k}{c}$

d. (d)  $d = \frac{1}{c}(\log_5 u + k)$

e. (e)  $d = \frac{k \log_5 u}{\log_5 c}$

5. Solve  $8^x = 4^{x^2-3}$ .

6. Solve  $\frac{e^x + 2e^{-x}}{3} = 1$ . FOR BONUS, verify your solution(s).

7. Express  $4 \ln x - 2(\ln x^3 + 4 \ln x)$  as a single logarithm.

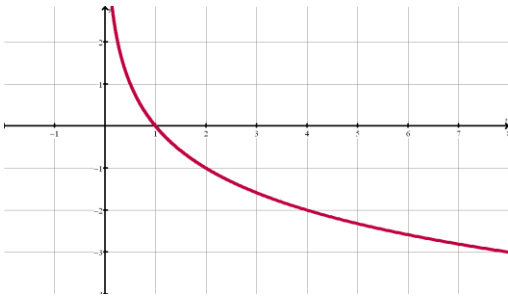
8. Find the domain of  $g(x) = \sqrt{e^x} \cdot \ln(4-x)$

9. Solve  $2(4^x) = 9(2^x) - 4$ .

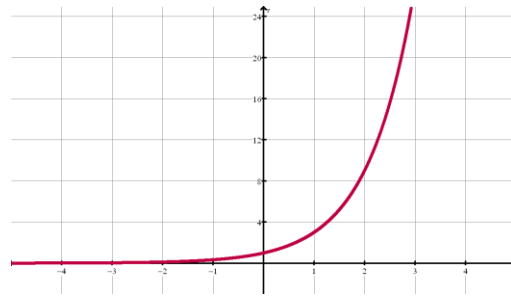
10. True or false. If  $-2(3^x)^4 + 8 = -4$ , then  $x = \frac{\ln 6}{\ln 81}$ . Show an algebraic rationale for your choice. No marks are awarded for simply selecting T or F.

PART B

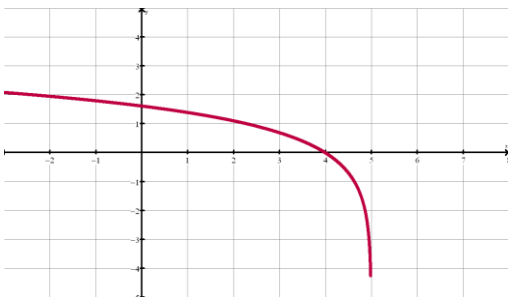
1. Match the following graphs with equations.



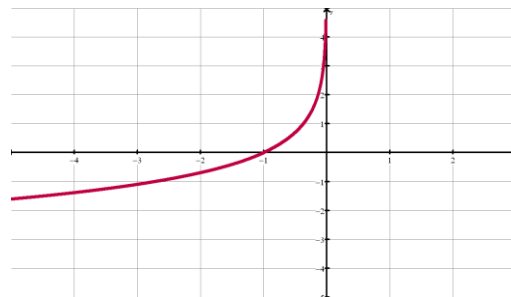
- (a)  $y = \log_{\frac{1}{2}}(x)$
- (b)  $y = \log_2(x)$
- (c)  $y = \left(\frac{1}{2}\right)^x$
- (d)  $y = (2)^x$
- (e)  $y = \log_2(-x)$



- (a)  $y = -3^{-x}$
- (b)  $y = 3^x$
- (c)  $y = -3^x$
- (d)  $y = 3^{-x}$
- (e)  $y = \log_3 x$



- (a)  $y = \ln(-x - 5)$
- (b)  $y = -\ln(5 - x)$
- (c)  $y = \ln(5 - x)$
- (d)  $y = \ln(5 + x)$
- (e)  $y = -\ln(x - 5)$



- (a)  $y = \ln(x)$
- (b)  $y = -\ln(-x)$
- (c)  $y = \ln(-x)$
- (d)  $y = e^{-x}$
- (e)  $y = -e^{-x}$

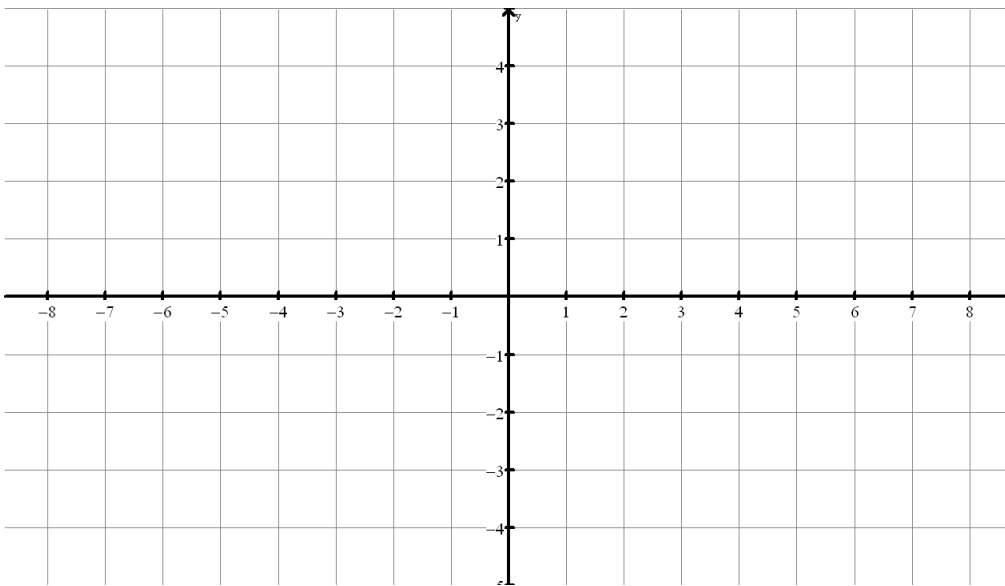
2. Solve the inequality  $\frac{\ln[(3-x)(x^2-4)]}{e^{1-x}} > 0$

## PART C

1. In an archeological dig, an animal bone was discovered. It was found that the amount of carbon-14 had decayed to  $\frac{1}{\sqrt{8}}$  of its original amount. If the half-life of carbon-14 is approximately 6000 years, how old is the skeleton?

2. Sketch a graph of  $f(x) = 2^x$  on the grid provided. Then, on the same grid, graph  $g(x) = 1 - \frac{1}{3} \cdot 2^{(2-x)}$ .

State the domain, range, asymptotes, x- and y-intercepts of the transformed function. State the applied transformations of the original  $y = f(x)$



## CALC ALLOWED

### PART A

1. An earthquake measures 4.6 on the Richter scale. A second earthquake is 1750 times as intense as the first earthquake. Find the Richter scale measure of the second earthquake.
2. Find the hydrogen ion concentration of a solution if its pH is 5.3.
3. An insect colony grows continuously from an initial population of 200 to 3000 in 2 months. How long will it take the population to grow to 50,000?
4. Find the equation of the inverse of  $f(x) = 2^{x+3} + 1$ . Verify your answer by composing the original function and its inverse  $\rightarrow$  i.e.  $f(f^{-1}(x))$  or  $f^{-1}(f(x))$

### PART B

1. You are given a system of equations defined by the two equations,  $f(x) = \log(x+3)$  and  $g(x) = e^{x-6x^2}$ 
  - a. Determine the domain of each function.
  - b. Find the intersection point(s) of the system of equations.
  - c. Include a sketch of the system in an appropriate window, highlighting the intersection point(s). State your window settings.
2. Solve  $\log_2(3x+2) + \log_2(x-9) = 5$ . Show necessary algebraic work.
3. A drug is administered for pain relief. The function  $A(t) = 90 - 52\ln(1+t)$  where  $0 \leq t \leq 4$ , gives the amount of drug remaining in the body after  $t$  hours. Express all final answers to 3 decimal places.
  - a. What is the initial amount of drug administered?
  - b. How much drug is in the body at 150 minutes?
  - c. When will 50 units of the drug be in the body?

### PART C

1. Mr. Santowski's class last year performed an experiment wherein they measured the temperature of my coffee over the course of a 85 minute period. The room temperature was  $26.8^\circ\text{C}$  and the data is included below:

Time	0	10	20	30	40	50	60	70	80
Temp	76.6	65.5	57.7	51.1	47	43.1	40.1	37.1	35.2

- a. Determine an appropriate model that fits the data and briefly justify your choice of models, given the context of the data.
- b. State any assumptions you made in developing your model.
- c. From your model, determine the temperature of the coffee after (i) 45 min, and (ii) 120 min.
- d. I like to drink my coffee when the temperature lies between 60 and 75 degrees. From your model, when can I drink my coffee?

2. asd
- 3.