Solving Systems of Linear Equations

Math 2 Honors - Santowski

Fast Five

- How would you algebraically, numerically, and graphically show whether the following statement is true or false
- The equations $2x - 5y = 12$ and $y = 0.4x - 2.4$ describe the same set of points.

Lesson Objectives

- 1. Students will define, classify and solve systems of linear equations.
- 2. Students will solve systems of linear equations algebraically using substitution and
- elimination.
- 3. Students will solve systems of linear equations graphically (by hand and using the TI-
- 84), and know that the solution refers to the point where the two lines intersect.
- 4. Students will check solutions to systems on the TI-84 and by hand.

Definitions

- A system of linear equations refers to a collection of linear equations involving the same set of variables
- A solution to a system of linear equations is an assignment of numbers to the variables such that all the equations are simultaneously satisfied

(A) Algebra Review – Substitution Method

- Systems of linear equations can be solved algebraically by using the SUBSTITUTION method
- Solve the system of equations defined by

$$\begin{cases} y = 2x - 5 \\ x + y = 1 \end{cases}$$

- To verify that your solution is correct, use BOTH equations
- Use the TI-84 to graph and solve the system

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(A) Algebra Review – Elimination Method

- Systems of linear equations can be solved algebraically by using the ELIMINATION method
- Solve the system of equations defined by

$$\begin{cases} 4x - 2y = 7 \\ x + 2y = 3 \end{cases}$$

- To verify that your solution is correct, use BOTH equations
- Use the TI-84 to graph and solve the system

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(A) Algebra Review – 2 Methods

- Explain the conditions under which either method (elimination or substitution) is the more "user friendly"
- Use elimination to solve the system of equations defined by

$$\begin{cases} 3^x - y = -1 \\ 2y - 9^{2x} = 4 \end{cases}$$

- And your point is .... ?

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(B) Classifying Systems

- Systems of equations can de defined as being either consistent or inconsistent
- If a system has AT LEAST one solution, the system is classified as being consistent
- If the system has NO solution, then the system is classified as being inconsistent

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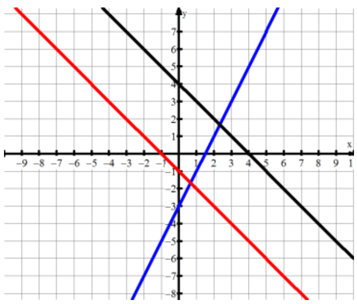
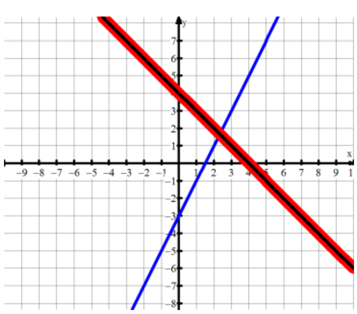
(B) Classifying Systems



(B) Classifying Systems

- Secondly, systems can also be classified as being either dependent or independent
- If a system has EXACTLY one solution, then the linear system is classified as independent
- If the system has INFINITELY MANY solutions, then the linear system is classified as dependent

(B) Classifying Systems



(C) Classifying Systems - Examples

- Use algebraic methods to solve and classify the linear system defined as

$$\begin{cases} y = x + 5 \\ y = -2x + 5 \end{cases}$$

- Use the TI-84 to graphically verify your solution and classification

(C) Classifying Systems – Examples

- Use algebraic methods to solve and classify the linear system defined as

$$\begin{cases} 3 = x + 3y \\ 1 = x - y \end{cases}$$

- Use the TI-84 to graphically verify your solution and classification

(C) Classifying Systems – Examples

- Use algebraic methods to solve and classify the linear system defined as

$$\begin{cases} y = 3x + 5 \\ -\frac{1}{2}y + \frac{3}{2}x = -1 \end{cases}$$

- Use the TI-84 to graphically verify your solution and classification

(C) Classifying Systems – Examples

- Use algebraic methods to solve and classify the linear system defined as

$$\begin{cases} y = -2x + 1 \\ 2 - 2y = 4x \end{cases}$$

- Use the TI-84 to graphically verify your solution and classification

(C) Classifying Systems – Examples

- Solve for k such that the system defined below is inconsistent

$$\begin{cases} 2x - y = 5 \\ 4x - k = 2y \end{cases}$$

- Can the system defined above be consistent? If so, how?

Linear Systems – Examples

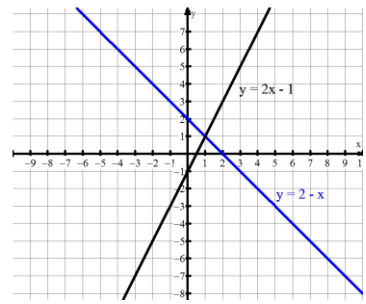
- Graph the given the system of linear equations defined as

$$\begin{cases} y = 2x - 1 \\ y = 2 - x \end{cases}$$

- NOW, EXPLAIN what the following inequality statement means and then state your solution

$$2x - 1 > 2 - x$$

Linear Systems – Examples



(C) Linear Systems – Examples

- Solve the linear system defined by

$$\begin{cases} Ax + By = C \\ Dx + Ey = F \end{cases}$$

(C) Linear Systems – Examples

- Solve the linear system defined below by "programming" an EXCEL spreadsheet, so that I can input any values for A,B,C,D,E,F and your spreadsheet will give me the solution.

$$\begin{cases} Ax + By = C \\ Dx + Ey = F \end{cases}$$

(D) In 3D??

- Since lines (Ax + By = C) are one dimensional objects drawn on a 2 dimensional plane (the grid/graph is called the Cartesian plane), can we move one dimension higher?
- What, then, would Ax + By + Cz = D represent?

(D) In 3D??

- What, then, would Ax + By + Cz = D represent ➔ it represents a 2 dimensional plane drawn in 3 dimensional space
- So just as we asked if/where do 2 lines in 2-space intersect, we can ask the same question in 3-space
- If/Where do 3 planes intersect??

(D) In 3D??

- Use a combination of elimination & substitution to solve the system defined by

$$\begin{cases} 3x+5y=-3 \\ 10y-2z=2 \\ x-z=0 \end{cases}$$

(E) Word Problems

- Linear systems are useful for applications in which the given information can be expressed as 2 linear equations ➔ "mixture" questions
- Mr S invests $17,000 into two stocks. Company A earns 5% pa and company B earns 10% pa. If I earned a total of $1450 last year, how much did I invest in each company?

Homework

- Ex 3.1 (p. 161) # 21-23, 37, 39, 40ab, 46, 52, 53
- Ex 3.2 (p. 169) # 19, 21, 23, 36, 37, 40, 43