

Lesson 4 – 1.8 – Solving Absolute Value Equations & Inequalities

Math 2 Honors - Santowski

Lesson Objectives

- Solve and graph absolute value equations
- Solve and graph absolute value inequalities
- Reinforce the equality of multiple representations of solving methods

(A) Absolute Value – The Concept Geometrically

- There are several ways to understand the concept of absolute value:
 - (i) the idea of an "undirected" distance, speed, force ...etc.... (think scalar quantities in Physics)
 - (ii) number line - the points +3 and -3 are both the same distance from 0. Hence, they have the same absolute value (think: distance from 0 (or from a rest position))
- So we can write this in the notation as $|-3| = 3$ and $|+3| = 3$
- So then the meaning behind the values is that absolute value seems to be a "mathematical operation" which takes either a positive or negative value as its "input" and returns a positive value as its "output"

(A) Absolute Value – The Concept Algebraically

- Absolute value can be seen as a function defined by the following piecewise function:

$$|x| = \begin{cases} -x & \text{if } x < 0 \\ x & \text{if } x \geq 0 \end{cases}$$

- So what does this mean:

$$|-3| = -(-3) = 3$$

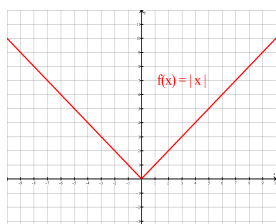
$$|3| = (3) = 3$$

(A) Absolute Value – The Concept Graphically

Our graph will start with a table of values

x	y
-10.00000	10.00000
-9.00000	9.00000
-8.00000	8.00000
-7.00000	7.00000
-6.00000	6.00000
-5.00000	5.00000
-4.00000	4.00000
-3.00000	3.00000
-2.00000	2.00000
-1.00000	1.00000
0.00000	0.00000
1.00000	1.00000
2.00000	2.00000
3.00000	3.00000
4.00000	4.00000
5.00000	5.00000
6.00000	6.00000
7.00000	7.00000
8.00000	8.00000
9.00000	9.00000
10.00000	10.00000

Here is a graph of $y = |x|$



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(A) Absolute Value – The Concept Graphically

- We can look at a graph of $y = |x|$ to solidify some of the geometric & algebraic ideas:
- Notice the graph of the function $y = x$ and how part of this graph (that part which is left of $x = 0$) seems to have been reflected over the y -axis
- (Hence the idea of if $x < 0$, then $|x| = -x$)

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(A) Absolute Value – The Concept

- So then the meaning behind these ideas is that absolute value seems to be a “mathematical operation” which takes either a positive or negative value as its “input” and returns a positive value as its “output”

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(B) Working With Absolute Value: Examples

- Simplify the following
 - (i) $|-8|$
 - (ii) $|0 - 6|$
- Simplify
 - (i) $|5 - 2|$
 - (ii) $|2 - 5|$
- evaluate $|-2|$, $|4|$, and $|8-5|$
- evaluate $|3| + |-5|$
- evaluate $-|-3| + 2|3| - 6|-5| - |-6| + 2|0|$

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(C) Equations with Absolute Value

- Solve and verify $|2x - 3| - 4 = 3$
- Solve and verify $-2|x/2 + 3| - 4 = -10$
- Solve and verify $|2x - 2| = x + 1$
- Solve and verify $|x + 1| = x - 3$
- Solve and verify $|x^2 - 4x - 5| = 7$

- Solve each of the equations GRAPHICALLY as well

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(C) Equations with 2 Absolute Values

- But what about the equation $|2x - 1| = |4x + 3|$

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(C) Equations with 2 Absolute Values

- Solve and verify $|2x - 1| = |4x + 3|$
- So here we have to consider various possibilities:
- The "input" of $(2x - 1)$ could be $+(2x - 1)$ or $-(2x - 1)$
- Likewise, the input of $(4x + 3)$ could be $+(4x + 3)$ or $-(4x + 3)$

- So then we could potentially set up 4 equations:
 - $+(2x - 1) = +(4x + 3)$
 - $-(2x - 1) = -(4x + 3)$
 - $+(2x - 1) = -(4x + 3)$
 - $-(2x - 1) = +(4x + 3)$
- But note that really equations #1 and #2 are the same, as are equations #3, #4
→ so I only really need 2 of these equations, say #1 and #3
- So now solve this equation ($x = -2, -1/3$) (Solution at [the following website](#))

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(D) Inequalities with Absolute Value

- Let's work through some algebraic solutions to inequalities with absolute value and simultaneously consider the graphic solutions.

- In each case, we will record our final solution in set and interval notation

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(D) Inequalities with Absolute Value

- Solve and verify:
 - $|2x - 4| < 10$
 - $|9x - 2| \leq 1$
 - $|2x - 3| > 7$
 - $-|7t + 10| \geq 3$
 - $|3x + 2| < 0$
 - $|4x + 15| < -2$
- Solve and verify:
 - $|2x - 4| < x - 2$
 - $|2x - 3| \geq -1/2x + 3$
 - $|x^2 - 4| > x + 2$

Internet links

- <http://www.youtube.com/watch?v=rC4bRSQetvQ>
- <http://tutorial.math.lamar.edu/Classes/Alg/SolveAbsValueIneq.aspx>
- http://www.wtamu.edu/academic/anns/mps/math/mathlab/col_algebra/col_alg_tut21_abseq.htm
- http://www.anlyzemath.com/Equations/Absolute_Value_Tutorial.html
- <http://www.purplemath.com/modules/solveabs1.htm>

(H) Homework

- p. 69 # 33-39 odds, 65
- p. 67 # 19-24, 53-61