2.6 – Special Functions

Math 2 Honors - Santowski

Lesson Objectives

- Define and graph piecewise functions, step functions, and absolute-value functions
- Use these special functions to review the following prior lesson objectives:
  - Evaluate functions
  - Analyze functions
  - Add, subtract, multiply, and divide functions
  - Find compositions of functions
  - Find the inverse of a function

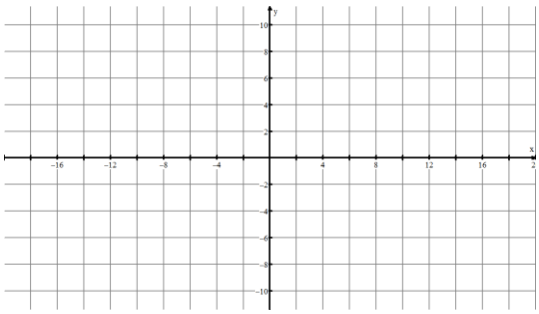
(A) Piecewise Functions

- The following function is called a piecewise function. WHY??
- Graph by preparing a table of values and analyze

$$f(x)=\begin{cases}2x-1 & \text{if } x<3\\ -x & \text{if } x\geq 3\end{cases}$$

- (a) state domain, range, and intercepts
- (b) NEW TERM: Is f(x) a continuous function?
- (c) Graph the inverse, $f^{-1}(x)$
- (d) Determine the domain and range of the inverse
- (e) is the inverse a function or not?

Grid to Use



(A) Piecewise Functions

- The following function is called a piecewise function. WHY??
- Graph by preparing a table of values and analyze

$$f(x)=\begin{cases} x^2-1 & \text{if } x<-2 \\ 2 & \text{if } -2\le x\le 2 \\ 1-x & \text{if } x>2 \end{cases}$$

- (a) state domain, range, and intercepts
- (b) NEW TERM: Is f(x) a continuous function?
- (c) Graph the inverse, $f^{-1}(x)$
- (d) Determine the domain and range of the inverse
- (e) is the inverse a function or not?

(A) Piecewise Functions

- The following function is called a piecewise function. WHY??
- Graph by preparing a table of values and analyze

$$f(x)=\begin{cases} -x & \text{if } x<0 \\ x & \text{if } x\ge 0 \end{cases}$$

- (a) state domain, range, and intercepts
- (b) NEW TERM: Is f(x) a continuous function?
- (c) Graph the inverse, $f^{-1}(x)$
- (d) Determine the domain and range of the inverse
- (e) is the inverse a function or not?

(B) The Absolute Value Function

- Recall that the absolute value function was defined as a piecewise function as you just reviewed on the previous slide
- If f(x) = |x|, then evaluate the following:
- (a) f(-1) + 5 f(-2) + 5
- (b) f(-1 + 5) f(-2 + 5)
- (c) -2f(-1) -2f(-2)
- If f(x) = |x|, then graph the following by using a table of values:
- (a) y = f(x) + 5
- (b) y = f(x + 5)
- (c) y = -2f(x)

(C) Step Functions

- One step function, the greatest integer function, is a function that takes an input and ROUNDS the input value DOWN to the nearest integral value:
- The notation is $f(x)=\lfloor x \rfloor$
- ex. of evaluations are:

$$f(3.2)=\lfloor 3.2 \rfloor =$$
$$f(3.5)=\lfloor 3.5 \rfloor =$$
$$f(3.9)=\lfloor 3.9 \rfloor =$$
$$f(3.9999)=\lfloor 3.9999 \rfloor =$$
$$f(-1.5)=\lfloor -1.5 \rfloor =$$

(C) Greatest Integer Function

- Prepare a table of values and graph $f(x)=\lfloor x \rfloor$
- Now graph the following, given that $f(x)=\lfloor x \rfloor$
- (a) y = |f(x)|
- (b) y = $f^{-1}(x)$

(C) Step Functions

- Another step function, a ceiling function, is a function that takes an input and ROUNDS the input value UP to the nearest integral value (i.e. Phone companies who charge on a per minute basis)
- The notation is $f(x)=\lceil x \rceil$
- ex. of evaluations are:

$f(3.2)=\lceil 3.2 \rceil =$

$f(3.5)=\lceil 3.5 \rceil =$

$f(3.9)=\lceil 3.99 \rceil =$

$f(4.001)=\lceil 4.001 \rceil =$

$f(-1.5)=\lceil -1.5 \rceil =$

(D) Incorporating Function Concepts

- Determine the equation for, state domain, evaluate y(-2.2) and then graph y(x), given the following four functions that are used to define y(x) are as follows:

$$f(x)=|x|,\quad g(x)=\lfloor x \rfloor,\quad h(x)=2x-6,\quad t(x)=2-x^2$$

$y(x)=f\circ t(x)$ $\qquad y(x)=\dfrac{f(x)}{x}$

$y(x)=t\circ g(x)$ $\qquad y(x)=h^{-1}\circ t^{-1}(x)$

$y(x)=h(x)+t(x)$ $\qquad y(x)=\dfrac{f(x)}{h(x)}$

$y(x)=f\circ h(x)$ $\qquad y(x)=\begin{cases} f(x) & x<3 \\ t(x) & x\geq 3 \end{cases}$

(D) Incorporating Function Concepts

- Are function operations associative??
- Use algebraic and graphic evidence to support your conclusions if

$$f(x)=|x|,\quad h(x)=2x-6,\quad t(x)=2-x^2$$

- (a) is addition? $(f(x)+t(x))+h(x)=?=f(x)+(t(x)+h(x))$
- (b) is multiplication? $(f(x)\times t(x))\times h(x)=?=f(x)\times(t(x)\times h(x))$
- (c) is composition? $(f(x)\circ t(x))\circ h(x)=?=f(x)\circ(t(x)\circ h(x))$

Homework

- p. 129 # 21-22, 26-31, 53-65 odds, 66