2.5 - The Inverse Function

Math 2 Honors - Santowski

Lesson Objectives

- Find the inverse of a function from numeric/tabular, graphic or algebraic data
- Use the horizontal line test to determine whether the inverse is a function
- Understand inverses as transformations
- Compose a function with its inverse to develop the identity function

Fast Five

- 1. You have the functions y = f(x) and y = g(x) as follows:

$$f(x) = x - 2 \text{ and } g(x) = x^2 - x + c$$

- (a) Determine the domain and range of y = g(x)
- 2. Now you will work with the function $y(x) = \frac{f(x)}{g(x)}$
- (a) Determine the value of c such that:
- (i) the domain of y(x) is infinite
- (ii) the domain of y(x) has only one restriction
- (iii) what would happen on the graph of y(x) if c = -6

(A) Inverses - The Concept

- Let's back to our input ➔ output notion for functions.
- If functions are nothing more than input/output operators, then the concept of an inverse has us considering how to go in reverse ➔ how do you get from the output back to the input?

(B) Inverses - An Example of the Concept

- Consider is the idea of a relationship between °F and °C ➔ the relationship says that if we know °C we can convert to °F by means of the simplified formula double the temperature in °C and add 30°.
- If we work with this simplified formula, we can generate ordered pairs, (or tables of graphs) ➔ Order pairs would include (-10,10), (0,30), (10,50), (20,70), (30,90)
- in our input/output formula idea we would have: input ➔ times 2 ➔ add 30 ➔ output
- if we wish to discuss the idea of an inverse, we would ask "how do we go from °F back to °C?"
- So to express the inverse, we would switch or reverse the ordered pairs or our input/output notion ➔ ordered pairs are(10,-10), (30,0), (50, 10), (70, 20), (90, 30) etc….
- Our input/output formula idea would have us: do WHAT??

(C) Definition of an Inverse

- A function is a correspondence or relationship between the elements of two sets of numbers, which may be expressed as set of ordered pairs, tables, graphs or formulas.
- If the elements of the ordered pairs or mappings are reversed, the resulting set of ordered pairs or mappings are referred to as the INVERSE.
- Another point worth noting: the domain of the original function now becomes the range of the inverse; likewise, the range of the original becomes the domain of the inverse.

(D) Notation of the Inverses

- The notation used for the inverses of functions is $f^{-1}(x)$.
- IMPORTANT NOTE: $f^{-1}(x)$ does not mean $(f(x))^{-1}$ or $1 / f(x)$.

(E) Inverses as Transformations

- Place the ordered pairs on the Cartesian plane and we see the relationship between the original ordered pairs and the transformed ordered pairs of the inverse
- The relationship that exists is that the original points are reflected in the line y = x.

(F) Determining Equations of Inverses

- Let's start with the ordered pairs of the linear relation as follows:
- (2,8), (4,9), (8,11) (-2,6), (-6,4)
- By considering our slope ➔ a rise of 2 over a run of 1 ➔ then use a point to find y-intercept of 7 ➔ the equation relating the ordered pairs is y = 0.5x + 7
- the ordered pairs of the inverse will be (8,2), (9,4), (11,8), (6,-2), (4,-6)
- By considering our slope ➔ a rise of 2 over a run of 1 ➔ then use a point to find the y-intercept of –14 ➔ the equation of the inverse is y = 2x - 14

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(F) Determining Equations of Inverses

- Let's start with the ordered pairs of the following relation:
- (0,-3), (1,-2), (2,1) (3,6), (4,13)
- Look for a pattern in the data ➔ can you find a mathematical relationship between x & y?
- the ordered pairs of the inverse will be (-3,0), (-2,1), (1,2), (6,3), (13,4)
- So can you find a mathematical relationship between x & y in this inverse relation?
- Alternatively, can you find some algebraic method of "undoing" your mathematical relationship from before?

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(F) Determining Equations of Inverses

- Let's start with the ordered pairs of the following relation:
- (0,2), (2,5), (4,17) (6,65), (8,257)
- Look for a pattern in the data ➔ can you find a mathematical relationship between x & y?
- the ordered pairs of the inverse will be (2,0), (5,2), (4,17), (65,6), (257,8)
- So can you find a mathematical relationship between x & y in this inverse relation?
- Alternatively, can you find some algebraic method of "undoing" your mathematical relationship from before?

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(F) Algebraic Method For Determining Equations

- How can I manipulate y = 2x + 7 to get y = 2x - 14, or the reverse, how can I manipulate y = 2x - 14 to get y = 2x + 7?
- Since forming the inverse involved switching the order of an ordered pairs (x←→y) then let's try this for the algebraic approach
- If y = ½ x + 7 ➔ then x = ½ y + 7 ➔ So x – 7 = ½ y
- And thus 2x – 14 = y which was the equation of our inversed ordered pairs
- Thus the algebraic method is to switch the x and y and then solve the resultant equation for y.

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(F) Algebraic Method For Determining Equations

- How can I manipulate $y = x^2 - 3$ to get $y = \sqrt{(x + 3)}$, or the reverse, how can I manipulate $y = \sqrt{(x + 3)}$ to get $y = x^2 - 3$?
- Since forming the inverse involved switching the order of an ordered pairs ($x \leftrightarrow y$) then let's try this for the algebraic approach

- If $y = x^2 - 3$ ➔ then $x = y^2 - 3$ ➔ So $x + 3 = y^2$
- And thus $y = \sqrt{(x + 3)}$ which was the equation of our inversed ordered pairs

- Thus the algebraic method is to switch the x and y and then solve the resultant equation for y.

(F) Algebraic Method For Determining Equations

- How can I manipulate $y = 2^x + 1$ to get the inverse?
- Since forming the inverse involved switching the order of an ordered pairs ($x \leftrightarrow y$) then let's try this for the algebraic approach

- If $y = 2^x + 1$ ➔ then $x = 2^y + 1$ ➔ So $x - 1 = 2^y$
- And now we need to "unexponentiate" 2^y ?????? But how???

- Thus the algebraic method is to switch the x and y and then solve the resultant equation for y.

(G) Examples

- Ex. Determine the equation for the inverse of $y = 4x - 9$. Draw both graphs and find the D and R of each.

- Ex. Determine the equation for the inverse of $y = 2x^2 + 4$. Draw both and find D and R of each.

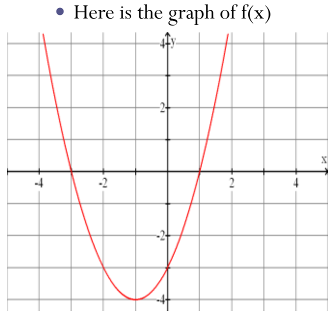
- Ex. Determine the eqn for the inverse of $y = 2 - \sqrt{(x+3)}$. Draw both and find the domain and range of each.

- Ex. Determine the eqn of the inverse of $y = x^2 + 4x - 2$. Draw both and find the domain and range of each.

(G) Examples

- ex 2. Consider a graph of the following data:
- 1. State do domain and range of f
- 2. Evaluate f(-2), f(0), $f^{-1}(1)$, $f^{-1}(-2)$
- 3. Graph the inverse
- 4. Is the inverse a function?
- 5. State the domain and range of $f^{-1}(x)$

- Here is the graph of f(x)



(G) Examples

- ex . The cost of renting a car for a day is a flat rate of $40 plus $0.10/km
- 1. Write a function r(d) to represent the total cost of a one day rental. State restrictions on domain and range.
- 2. Find the equation of the inverse. What does the equation of the inverse represent?
- 3. Give an example of how the inverse could be used?

(G) Examples

- ex . If an object is dropped from a height of 80 m, its height above the ground in meters is given by
- $h(t) = -5t^2 + 80$
- 1. Graph the function
- 2. Find and graph the inverse
- 3. Is the inverse a function
- 4. What does the inverse represent?
- 5. After what time is the object 35 m above the ground?
- 6. How long does the object take to fall?

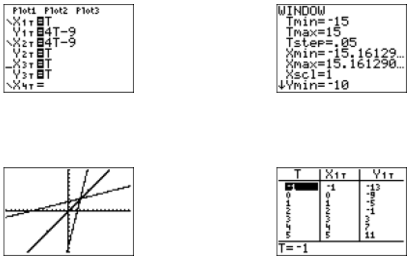
(H) Composing with Inverses

- Let $f(x) = 2x - 7$.
- Determine the inverse of $y = f(x)$
- Graph both functions on a grid/graph
- Fold the grid/graph upon the line $y = x$. What do you observe? Why?
- What transformation are we considering in this scenario?
- Now compose as follows $f o f^{-1}(x)$ and $f^{-1} o f(x)$. What do you notice?
- How is this related to our graph folding exercise?

(H) Composing with Inverses

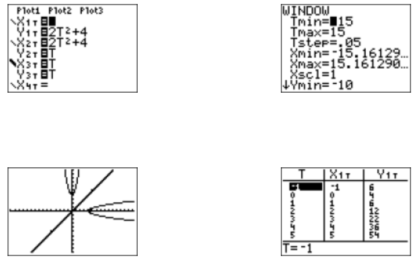
- Now let $f(x) = x^2 + 2$.
- Determine the inverse of $y = f(x)$
- Graph both functions on a grid/graph
- Fold the grid/graph upon the line $y = x$. What do you observe? Why?
- What transformation are we considering in this scenario?
- Now compose as follows $f o f^{-1}(x)$ and $f^{-1} o f(x)$. What do you notice?
- How is this related to our graph folding exercise?

Inverses on the TI-84 & Parametric Mode



21 Math 2 Honors - Santowski 8/25/2009

Inverses on the TI-84 & Parametric Mode



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(H) Internet Links

- Inverse Function Definition - Interactive Applet from AnalyzeMath
- Inverse Function - Interactive Tutorial from AnalyzeMath
- Inverse Functions Lesson - I from PurpleMath

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(H) Homework

- Math 2 Hon - p. 122 # 16-18, 27-29, 37-39, 46-51

24 Math 2 Honors - Santowski 8/25/2009