

Lesson 7 - Algebraic Combinations of Functions

Math 2 Honors - Santowski

1

Math 2 Honors

8/20/2009

Fast Five

- Graph $f(x) = 1 - x^2$ on the TI-84
- Graph $g(x) = \frac{1}{2}x - 4$ on the TI-84
- Graph $(f/g)(x)$ on the TI-84 and explain what happens at $x = 8$ and why.
- If $g(x)$ were defined as $g(x) = ax + b$, what is the domain of $(f/g)(x)$
- What if $g(x) = ax^2 - c$?

2

Math 2 Honors

8/20/2009

Lesson Objectives

- State the domain and range of a relation, and state whether it is a function
- Evaluate functions
- Add, subtract, multiply, and divide functions and analyze in terms of domains and ranges

3

Math 2 Honors

8/20/2009

(A) Investigation - Defining the Functions

- Let $f(x) = 1 - x^2$
- First we will work with an unrestricted domain; then secondly we will restrict our domain to $[-3, 4]$
- Let $g(x) = \frac{1}{2}x - 4$
- First we will work with an unrestricted domain; then secondly we will restrict our domain to $[-2, 3]$
- In all our investigations, we will see how new functions are created from simple functions that we have seen previously

4

Math 2 Honors

8/20/2009

(B) Operations with Functions - Addition/Subtraction

- Graph the following functions on the TI-84 or graphing software. Prepare a table of values for each new function. Discuss the domain, range, and intercepts of the newly formed functions
- (i) $f(x) + g(x) = (f + g)(x)$
- (ii) $f(x) - g(x) = (f - g)(x)$
- (iii) $g(x) - f(x) = (g - f)(x)$
- (iv) $|f(x) - g(x)|$ (absolute value)

5

Math 2 Honors

8/20/2009

(C) Operations with Functions - Multiplication/Division

- Graph the following functions on the TI-84 or graphing software. Prepare a table of values for each new function. Discuss the domain, range, and intercepts of the newly formed functions
- (i) $(fg)(x)$
- (ii) $(gf)(x)$
- (iii) $(f \div g)(x) = (f/g)(x)$
- (v) $(g \div f)(x) = (g/f)(x)$

6

Math 2 Honors

8/20/2009

(D) Operations with Functions - Reciprocals

- Graph the following functions on the TI-84 or graphing software. Prepare a table of values for each new function. Discuss the domain, range, asymptotes, and intercepts of the newly formed functions
- (i) $1/f(x)$
- (ii) $1/g(x)$

7

Math 2 Honors

8/20/2009

(E) Composition of Functions - An Example

- The following examples will be various ways of representing the composition of functions
- ex 1. Andrew earns a daily wage of \$20/h plus \$15/d for travel expenses.
- We can write his daily earnings as an equation as $Earnings = 20h + 15$.
- However, Andrew also pays union fees at 2.5% of his daily earnings, which we can write as the equation $Fees = 0.025 \times (\text{daily earnings})$
- We can demonstrate with a table of values

Hours worked per day	Daily Earnings	Union Fees Paid
2	$20(2) + 15 = 55$	$0.025(55) = 1.375$
5	$20(5) + 15 = 115$	$0.025(115) = 2.875$

8

Math SL1 - Santowski

8/20/2009

(E) Composition of Functions – AnExample

- What we see is that the one function value (daily earnings or E) is being substituted into the second function (Fees = $0.025 \times$ daily earnings) in order to generate the value for the union fees.
- We can generate a direct formula for the union fees by substituting the earnings function into the Fees function as follows: Fees = $0.025(20h + 15)$.
- Hence, the Fees function is called a composed function as Fees(daily earnings) = $0.025 \times$ daily earnings

9

Math SL1 - Santowski

8/20/2009

(E) Composition of Functions – Example #2

- ex 2. We will now define f and g as follows:
- $f = \{(3,2), (5,1), (7,4), (9,3), (11,5)\}$
- $g = \{(1,3), (2,5), (3,7), (4,9), (5,10)\}$
- We will now compose these two functions as follows:
- (i) we will substitute g into f which we will notate in two ways: $f(g(x))$ or as $f \circ g(x)$.
- We will evaluate $f \circ g(3)$ (or $f(g(3))$) \rightarrow ?????

10

Math SL1 - Santowski

8/20/2009

(E) Composition of Functions – Example #2

- ex 2. We will now define f and g as follows:
- $f = \{(3,2), (5,1), (7,4), (9,3), (11,5)\}$
- $g = \{(1,3), (2,5), (3,7), (4,9), (5,10)\}$
- We will now compose these two functions as follows:
- (i) we will substitute g into f which we will notate in two ways: $f(g(x))$ or as $f \circ g(x)$.
- We will evaluate $f \circ g(3)$ (or $f(g(3))$) \rightarrow we will substitute a specific g function value into f , that of $g(3)$.
- The g function value at $x = 3$ is 7 (i.e. $g(3) = 7$)
- So now we evaluate f at the new value of $x = 7$ \rightarrow we look at our f data and see that $f(7) = 4$
- (ii) evaluate $f \circ g(1)$
- (iii) evaluate $f \circ g(5)$ and see what happens \rightarrow why?
- (iv) evaluate $g \circ f(9)$ and $g \circ f(7)$ and $g \circ f(1)$

11

Math SL1 - Santowski

8/20/2009

(E) Composition of Functions – Example #3

- We can define f and g differently, this time as formulas:
- $f(x) = x^2 - 3$
- $g(x) = 2x + 7$
- We will try the following:
- (i) $f \circ g(3)$ or $f \circ g(3)$
- (ii) $g \circ f(3)$ or $g \circ f(3)$
- (ii) $f \circ g(x)$ and $g \circ f(x)$
- (ii) evaluate $f \circ g(5)$
- (iii) evaluate $g \circ f(9)$ and $g \circ f(7)$ and $g \circ f(1)$
- Repeat if $f(x) = -2|x^2 + x - 1|$ and $g(x) = 2^x$

12

Math SL1 - Santowski

8/20/2009

(E) Composition of Functions – New Domains

- When we create a new function from 2 or more functions (as in composing 2 functions), we must consider the domain of the new function
- So domain restrictions to remember from previous courses:
 - Square root functions → we take square roots of positive numbers
 - Reciprocal functions → denominators cannot be zero
 - Logarithmic functions → you cannot take the log of a negative value
 - Exponential, polynomial, absolute value, trigonometric functions → unrestricted domains
- Now go back to the previous slide and state the domain of the new functions you created

13

Math SL1 - Santowski

8/20/2009

(F) Operations with Functions - Inverses

- Graph the following functions on the TI-84 or graphing software. Prepare a table of values for each new function. Discuss the domain, range and intercepts of the newly formed functions
 - (i) $g^{-1}(x)$
 - (ii) $f^{-1}(x)$

14

Math 2 Honors

8/20/2009

(F) Internet Links

- [College Algebra Tutorial Operations with Functions](#)
- [Polynomial Functions from AnalyzeMath](#)

15

Math 2 Honors

8/20/2009

(G) Homework

- p. 115 # 16-17, 25-29, 37-45 odds, 53-59 odds
- Complete Assignment #1
- [Link to Assignment #1](#)

16

Math 2 Honors

8/20/2009