Lesson 6a – Introduction to Functions: Concepts and Notations

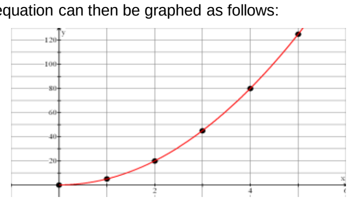
Math 2 Honors - Santowski

(A) Concept of Functions & Relations

- In many subject areas, we see relationships that exist between one quantity and another quantity.
  - ex. Galileo found that the distance an object falls is related to the time it falls.
  - ex. distance traveled in car is related to its speed.
  - ex. the amount of product you sell is related to the price you charge.
- All these relationships are classified mathematically as **Relations**.
- A **Relation** then is simply a set of ordered pairs or data points

(B) Representation of Functions & Relations

- Relations can be expressed using ordered pairs i.e. (0,0), (1,5), (2,20), (3,45), (4,80), (5,125)
- The relationships that exist between numbers are also expressed as equations: $s = 5t^2$
- This equation can then be graphed as follows:



(C) Terminology of Functions & Relations

- Two terms that we use to describe the relations are **domain** and **range**.
- Domain refers to the set of all the first elements, input values, independent variable, etc.. of a relation, in this case the time. We will express domain in set notation and in interval notation
- Range refers to the set of all the second elements, output values, dependent values, etc... of the relation, in this case the distance. We will express the range in set notation and in interval notation

(D) Functions - The Concept

- A **function** is a special relation in which each **single** domain element corresponds to exactly **one** range element.
- In other words, each input value produces one unique output value
- Or put another way, each value of the independent variable produces or causes one unique value of the dependent variable

(D) Functions - The Concept - Examples

- Make a mapping diagram for the relation {(-2,1), (-2,3), (0,3), (5,4)} and determine whether or not the relation is a function. Give a reason for your answer.
- State the domain and range of the following relation. Is the relation a function?
  {(–3, 5), (–2, 5), (–1, 5), (0, 5), (1, 5), (2, 5)}

(D) Functions - The Concept - Examples

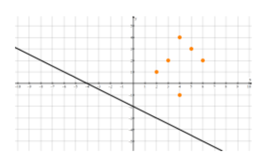
- A relation is defined by the set {(-1,2), (3,0), (5,2)}.
- (a) Sketch the set on a Cartesian plane and label the ordered pairs
- (b) Make a mapping diagram of this relation
- (c) State the domain of this relation
- (d) State the range of this relation

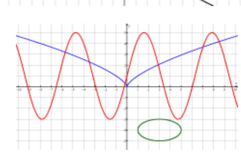
(D) Functions - The Concept - Examples

- Given the following table of values or graphs, determine the domain, range and determine whether or not the relation is a function

1	4	7	2	5	1
1.2	-3.4	2	-3	1.2	5

10	20	30	40	50	60
-9	-15	-18	-18	-15	-9

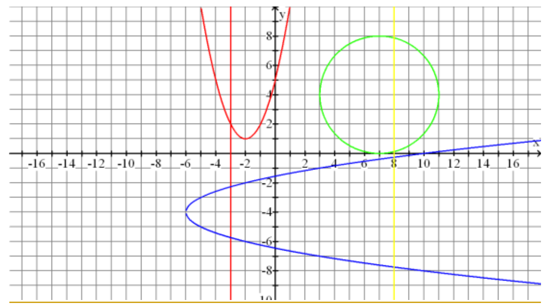




(E) Functions - Vertical Line Test

- To determine whether or not a relation is in fact a function, we can draw a vertical line through the graph of the relation.
- If the vertical line intersects the graph more than once, then that means the graph of the relation is not a function.
- If the vertical line intersects the graph once then the graph shows that the relation is a function.
- See the diagram on the next slide

(F) Functions - Vertical Line Test



(G) Functions - the Notation f(x)

- We have written equations in the form $y = 2x + 5$ or $y = 3x^2 - 4$.
- These equations describe the relationship between x and y, and so they describe relations ➔ since each x produced a unique y value, they are also functions
- Therefore we have another notation or method of writing these equations of functions.
- We can rewrite $y = 2x + 5$ as $f(x) = 2x + 5$
- We can rewrite $y = 3x^2 - 4$ as $g(x) = 3x^2 – 4$.

(H) Working with Function Notation

- For the function defined by $f(t) = 3t^2 - t + 4$, evaluate f(4):
- $f(4) = 3(4)^2 – (4) + 4 = 48 – 4 + 4 = 48$
- So notice that t = 4 is the “input” value (or the value of independent variable) and 48 is the “output” value (or the value of the dependent variable)
- So we can write f(4) = 48 or in other words, 48 (or f(4)) is the “y value” or the “y co-ordinate” on a graph
- So we would have the point (4,48) on a graph of t vs f(t)

(H) Working with Function Notation

- ex. For the function defined by $b(t) = 3t^2 - t + 3$, find:
- (a) b(-2) (b) b(0.5) (c) b(2) (d) b(t - 2)
- ex. For the function defined by $f(x) = +\sqrt{9 - x^2}$ graph it and then find new equations and graph the following:
- (a) f(3) (b) f(-2) (c) f(4) (d) f(a)
- ex. For the function defined by w(a) = 4a - 6, find the value of ***a*** such that w(a) = 8

(E) Functions and the TI-84

- We can graph functions & evaluate function values from the graph and also from the table of values

(G) Functions - the Notation f(x)

- Other notations with functions and ordered pairs given the function f(x) = 2x – 5
- (x,y)
- (x,f(x))
- (x, 2x – 5)

input	Output
6	2(6) – 5 = 7
-2	2(-2) – 5 =
√3	2( ) – 5 =
x	2( ) – 5 =
x	f(x)
x	y

(I) Internet Links

- College Algebra Tutorial on Introduction to Functions - West Texas A&M
- College Algebra Tutorial on Graphs of Functions Part I - from West Texas A&M
- Functions Lesson - I from PurpleMath
- Functions Lesson - Domain and Range from PurpleMath
- Functions from Visual Calculus
- Domains of Functions from Visual Calculus

(J) Homework

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