Lesson 48 – Solving Trigonometric Equations

Math 2 Honors - Santowski

(A) Review - Solving Equations

- The general strategy to ALGEBRAICALLY solving ANY equations is to:
- (a) Isolate the “base” function containing the unknown
- (b) Isolate the unknown by using the inverse of the base function

(A) Review - Solving Equations

- And a second alternative to solving ALGEBRAICALLY is to solve GRAPHICALLY by either:
- (a) looking for an intersection point for f(x) = g(x)
- (b) looking for the zeroes/roots of a rearranged eqn in the form of f(x) – g(x) = 0

(A) Examples

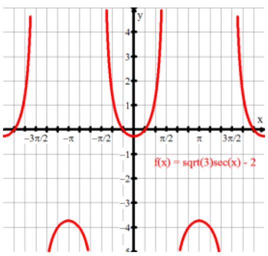
- Here are some easy examples to start with
- Let’s use the domain -2π ≤ x ≤ 2π and then work to an infinite domain

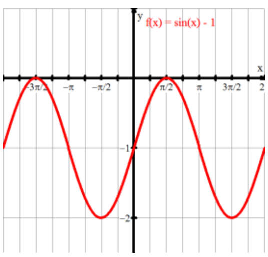
(a) $\sin(x) - 1 = 0$ (b) $\sqrt{3}\sec(x) - 2 = 0$

(c) $\tan(x) - 1 = 0$ (d) $2\sin(x) + \sqrt{3} = 0$

(e) $\sin(2x) = \frac{1}{2}$ (f) $\sqrt{2}\csc(4x) = -2$

(A) Examples - Graphically



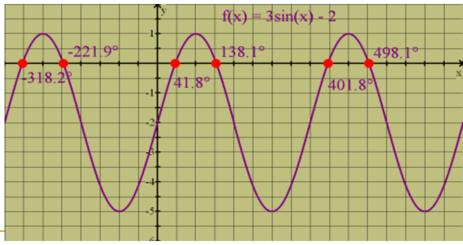


(B) Examples – With Calculator

- Solve the equation 3sin(x) – 2 = 0
- We can rearrange as sin(x) = 2/3 so x = $\sin^{-1}(2/3)$ giving us 41.8° (and the second angle being 180° - 41.8° = 138.2°)
- Note that the ratio 2/3 is not one of our standard ratios corresponding to our “standard” angles (30,45,60), so we would use a calculator to actually find the related acute angle of 41.8°

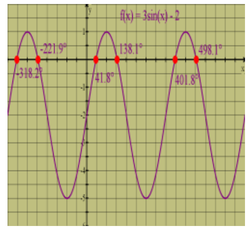
(B) Examples – With Calculator

- We can now solve the equation $3\sin(x) - 2 = 0$ by graphing $f(x) = 3\sin(x) - 2$ and looking for the x-intercepts



(B) Examples – With Calculator

- Notice that there are 2 solutions within the limited domain of $0° \leq \theta \leq 360°$
- However, if we expand our domain, then we get two new solutions for every additional period we add
- The new solutions are related to the original solutions, as they represent the positive and negative co-terminal angles
- We can determine their values by simply adding or subtracting multiples of 360° (the period of the given function)

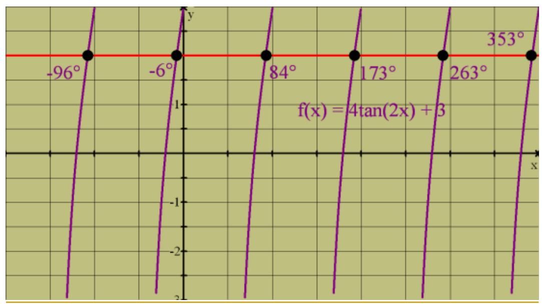


(B) Examples – With Calculator

- Solve $4\tan(2x) + 3 = 2$
- Again, we can set it up algebraically as $\tan(2x) = -1/4$ and thus $(2x) = \tan^{-1}(-1/4)$ so $x = ½ \tan^{-1}(-1/4)$
- So thus x = ½ of 166° = 84°
- and x = ½ of 346° = 173°

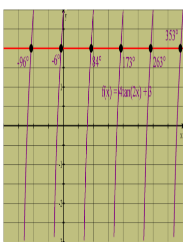
- To set it up graphically, we will make one minor change: we have two graphing options ➔ we can graph $f(x) = 4\tan(2x) + 1$ and find the x-intercepts OR we can graph $f(x) = 4\tan(2x) + 3$ and find out where $f(x) = 2$ ➔ to do this, we will simply graph $y = 2$ as a second equation and find out where the two graphs intersect

(B) Examples – With Calculator



(B) Examples – With Calculator

- Notice that there are 2 solutions within the limited domain of $0° \leq \theta \leq 180°$ (NOTE: the period of a regular tan function is 180°)
- However, if we expand our domain, then we get two new solutions for every additional period we add
- The new solutions are related to the original solutions, as they represent the positive and negative co-terminal angles
- We can determine their values by simply adding or subtracting multiples of 90° (the period of the given function ➔ the effect of the 2x in tan(2x) is a horizontal compression by a factor of 2, so we have reduced the period by a factor of 2)



(C) Quadratic Trigonometric Equations

- Quadratic trig eqns contain terms like $\sin^2(\theta)$, $\cos^2(\theta)$
- Recall the Pythagorean identity $(\sin^2(\theta) + \cos^2(\theta) = 1)$
- Recall how to factor simple trinomials like $x^2 + 2x - 35 = 0$ ➔ $(x + 7)(x - 5) = 0$
- Recall how to factor difference of square trinomials like $4x^2 - 25 = 0$ ➔ $(2x - 5)(2x + 5) = 0$
- Recall how to factor trinomials in the form of $3x^2 - x - 4 = 0$ ➔ using decomposition or guess & check ➔ $(3x - 4)(x + 1) = 0$

(C) Quadratic Trigonometric Equations

- Solve $2\cos^2(\theta) = 1$ if $0° \le \theta \le 360°$

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- Solve $2\cos^2(\theta) = 1$ if $0° \le \theta \le 360°$

$$2\cos^2(\theta) = 1$$
$$\cos^2(\theta) = \tfrac{1}{2}$$
$$\cos(\theta) = \pm \tfrac{1}{\sqrt{2}}$$
$$\therefore \theta = \cos^{-1}\left(\pm \tfrac{1}{\sqrt{2}}\right)$$
$$\theta = 45°, 135°, 225°, 315°$$

(C) Quadratic Trigonometric Equations

- Solve $\cos^2(x) + 2\cos(x) = 0$ for $0 \le x \le 2\pi$

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- Solve $\cos^2(x) + 2\cos(x) = 0$ for $0 \le x \le 2\pi$

$$\cos^2(x) + 2\cos(x) = 0$$
$$\cos(x) \times (\cos(x) + 2) = 0$$
$$(i) \therefore \cos(x) = 0$$
$$x = \cos^{-1}(0)$$
$$x = \frac{\pi}{2}, \frac{3\pi}{2}$$
$$(ii) \therefore \cos(x) = -2$$
$$x = \cos^{-1}(-2)$$
$$x \notin R$$

(C) Quadratic Trigonometric Equations

- Solve $2\cos^2(x) - 3\cos(x) + 1 = 0$ for $0 \le x \le 2\pi$

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- Solve $2\cos^2(x) - 3\cos(x) + 1 = 0$ for $0 \le x \le 2\pi$

$$2\cos^2(x) - 3\cos(x) + 1 = 0$$
$$(2\cos(x) - 1)(\cos(x) - 1) = 0$$
$$(i) \therefore 2\cos(x) - 1 = 0$$
$$\cos(x) = \tfrac{1}{2}$$
$$x = \tfrac{\pi}{3}, \tfrac{5\pi}{3}$$
$$(ii) \therefore \cos(x) = 1$$
$$x = 0, 2\pi$$

(C) Quadratic Trigonometric Equations

- Solve $2\cos^2(x) - \sin(x) - 1 = 0$

(C) Quadratic Trigonometric Equations

- Solve $2\cos^2(x) - \sin(x) - 1 = 0$ ➔ notice we have both sin(x) and cos(x) in the equation ➔ so use the Pythagorean identity to make changes
- The equation becomes $2(1 - \sin^2(x)) - \sin(x) - 1 = 0$
- So this is now $-2\sin^2(x) - \sin(x) + 1=0$ or we can make it $0 = 2\sin^2(x) + \sin(x) - 1$ which we can now factor and solve on $0° \leq x \leq 360°$
- $(2\sin(x) - 1)(\sin(x) + 1) = 0$
- So $2\sin(x) - 1 = 0°$, meaning $x = 30°$ and $150°$
- And $\sin(x) + 1 = 0°$, meaning $x = 270°$

(D) Other Examples

- Solve for x:

$$(a)\ \sin(x) - \sqrt{3}\cos(x) = 0$$
$$(b)\ \sin(2x) = \sin(x)$$
$$(c)\ \cos\left(3 + \frac{\pi}{6}\right) = \frac{\sqrt{3}}{2}$$
$$(d)\ 4\sin^{x}(x) = 3$$
$$(e)\ 2\sec^2(x) = 1$$

Homework

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