Lesson 47 – Double Angle & Half Angle Identities

Math 2 Honors - Santowski

(A) Review

- List the six new identities that we call the addition subtraction identities

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$$\sin(A+B) = \sin A \cos B + \sin B \cos A$$
$$\sin(A-B) = \sin A \cos B - \sin B \cos A$$

$$\cos(A+B) = \cos A \cos B - \sin A \sin B$$
$$\cos(A-B) = \cos A \cos B + \sin A \sin B$$

$$\tan(A+B) = \frac{\tan A + \tan B}{1 - \tan A \tan B}$$
$$\tan(A-B) = \frac{\tan A - \tan B}{1 + \tan A \tan B}$$

(B) Using the Addition/Subtraction Identities

- We can use the new identities to develop new identities:
- Develop a new identity for:
- (a) sin(2x)
- (b) cos(2x)
- (c) tan(2x)

(B) Using the Addition/Subtraction Identities

- We can use the new identities to develop new identities:
- Develop a new identity for:
- (a) sin(2x) = 2 sin(x) cos(x)
- (b) cos(2x) = $\cos^2(x) - \sin^2(x)$
- (b) cos(2x) = $\cos^2(x) - (1 - \cos^2(x)) = 2\cos^2(x) - 1$
- (b) cos(2x) = $(1 - \sin^2(x)) - \sin^2(x) = 1 - 2\sin^2(x)$
- (c) tan(2x) =

(C) Proofs with Double Angle Formulas

- Prove the following identities:

(a) $$\frac{\cos 2x + 1}{\sin x} = \cot x$$

(b) $$\frac{\sin 2x}{\sin x} = 4\cos x - \frac{\cos 2x + 1}{\cos x}$$

(D) Using the Double Angle Formulas

- (a) If sin(x) = 21/29, where 0º ≤ x ≤ 90º, evaluate: (i) sin(2x), (ii) cos(2x), (iii) tan(2x)
- (b) SOLVE the equation cos(2x) + cos(x) = 0 for 0º ≤ x ≤ 360º
- (c) Solve the equation sin(2x) + sin(x) = 0 for -180º ≤ x ≤ 540º.
- (d) Write sin(3x) in terms of sin(x)
- (e) Write cos(3x) in terms of cos(x)

(E) Half Angle Identities

- Start with the identity cos(2x) = 1 – 2sin²(x)
- Isolate sin²(x) ➔ this is called a "power reducing" identity ➔ Why?
- Now, make the substitution x = θ/2 and isolate sin(θ/2)
- Start with the identity cos(2x) = 2cos²(x) - 1
- Isolate cos²(x) ➔ this is called a "power reducing" identity ➔ Why?
- Now, make the substitution x = θ/2 and isolate cos(θ/2)

(E) Half Angle Identities

- So the new half angle identities are:

$$\sin\left(\frac{\theta}{2}\right)=\pm\sqrt{\frac{1-\cos\theta}{2}}$$
$$\cos\left(\frac{\theta}{2}\right)=\pm\sqrt{\frac{1+\sin\theta}{2}}$$

(F) Using the Half Angle Formulas

- Prove the identity:

$$\text{(a)}\ \sin^2\left(\frac{x}{2}\right)=\frac{\sin^2 x}{2+2\cos x}$$
$$\text{(b)}\ 2\sec(x)\cos^2\left(\frac{x}{2}\right)=1+\sec(x)$$

(F) Using the Half Angle Formulas

- (a) If sin(x) = 21/29, where 0º ≤ x ≤ 90º, evaluate: (i) sin(x/2), (ii) cos(x/2)
- (b) develop a formula for tan(x/2)
- (c) Use the half-angle formulas to evaluate:

$$\text{(a)}\ \sin\left(\frac{\pi}{12}\right)\qquad \text{(b)}\ \cos\left(\frac{7\pi}{12}\right)$$
$$\text{(c)}\ \sin(112.5°)\qquad \text{(d)}\ \cos(67.5°)$$
$$\text{(e)}\ \sin(225°)$$

(F) Homework