Lesson 44 – Inverse Trig Functions

Math 2 Honors - Santowski

Lesson Objectives

- (1) synthesize their knowledge of inverse functions and the three primary trig ratios in order to introduce, define and understand the nature of the 3 inverse trig functions
- (2) evaluate inverse trig expressions withand without a GDC
- (3) graph and analyze the graphs of the y = $\sin^{-1}(x)$, y = $\cos^{-1}(x)$, y = $\tan^{-1}(x)$

(A) Review of Inverse Functions

- Notation for inverse functions ➔ y = $f^{-1}(x)$ ➔ NOTE that $f^{-1}(x) \neq \frac{1}{f(x)}$
- If f(x) is a 1:1 function, then the domain of f(x) is the range of $f^{-1}(x)$ and likewise, the range of f(x) is the domain of $f^{-1}(x)$
- mathematically, f(a) = b, then $f^{-1}(b) = a$
- Q ➔ in what way is y = $f^{-1}(x)$ a transformation of y = f(x)?
- Q ➔ What is the purpose of an inverse function??

(A) Review of Inverse Functions

- (a) if f(x) = x^2 then $f^{-1}(x)$ =
- (b) if f(x) = √x, then $f^{-1}(x)$ =
- (c) if f(x) = b^x, then $f^{-1}(x)$ =
- (d) if f(x) = $\log_b x$, then $f^{-1}(x)$ =
- (e) so if f(x) = sin(x), then $f^{-1}(x)$ =
- (f) and if f(x) = $\cos^{-1}(x)$, then $f^{-1}(x)$ =

(A) Review of Inverse Functions

- Q ➔ What does the term 1:1 function mean?
- Q ➔ What is the significance of a function being 1:1 in the context of inverse functions?
- Q ➔ Is f(x) = x^2 a 1:1 function?
- Q ➔ How did we "resolve" this issue with the function f(x) = x^2?

(A) Review of Inverse Functions

- (a) Graph the function f(x) = x^2
- (b) Graph the inverse of f(x)
- (c) Graph the inverse function of f(x)
- (d) Now do the same for g(x) = sin(x). Graph g(x) = sin(x)
- (e) Graph the inverse of g(x)
- (f) Graph the inverse function of g(x)

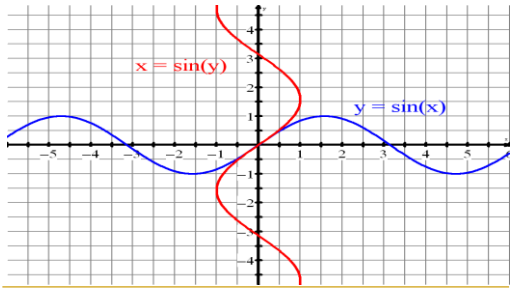
(B) Restricted Domains

- (a) How would you restrict the domain of g(x) = sin(x) so that its inverse is a function? Graphically justify your choice.
- (b) How would you restrict the domain of g(x) = cos(x) so that its inverse is a function? Graphically justify your choice.
- (c) How would you restrict the domain of g(x) = tan(x) so that its inverse is a function? Graphically justify your choice.

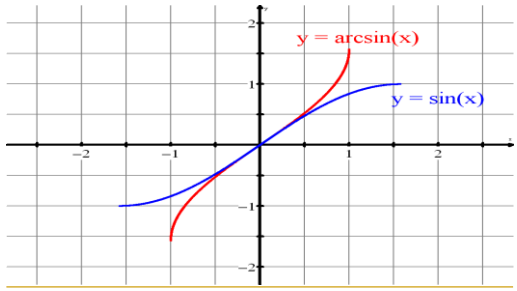
(B) Restricted Domains

- (a) How would you restrict the domain of g(x) = sin(x) so that its inverse is a function? Use your calculator to justify your choice. (numerically & graphically)
- (b) How would you restrict the domain of g(x) = cos(x) so that its inverse is a function? Use your calculator to justify your choice. (numerically & graphically)
- (c) How would you restrict the domain of g(x) = tan(x) so that its inverse is a function? Use your calculator to justify your choice. (numerically & graphically)

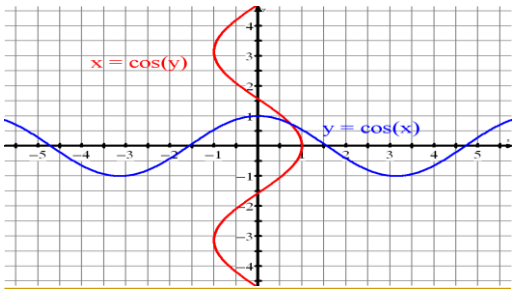
(C) Graphs of Trig Inverse Functions: $y = \sin^{-1}(x)$



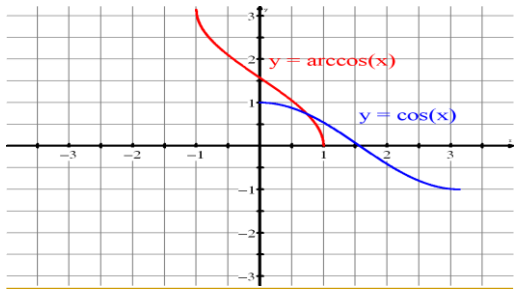
(C) Graphs of Trig Inverse Functions: $y = \sin^{-1}(x)$



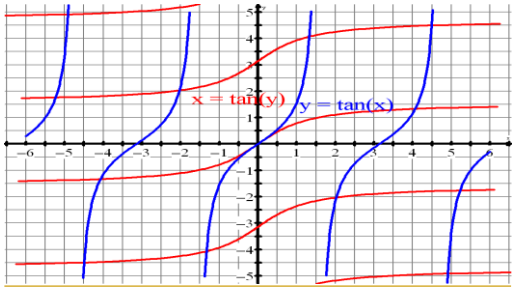
(C) Graphs of Trig Inverse Functions: $y = \cos^{-1}(x)$



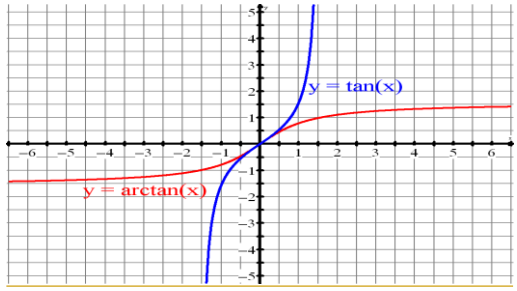
(C) Graphs of Trig Inverse Functions: $y = \cos^{-1}(x)$



(C) Graphs of Trig Inverse Functions: $y = \tan^{-1}(x)$



(C) Graphs of Trig Inverse Functions: $y = \tan^{-1}(x)$



(D) Solving Trig Equations

- (a) Solve sin(x) = 0.5
- (b) Solve $x = \sin^{-1}(0.5)$
- (c) Are your solutions for Q(a) and Q(b) the same or different? Explain.

(D) Solving Trig Expressions

- (a) Evaluate the following:

(*i*) $\sin\left(\frac{\pi}{3}\right) =$ (*ii*) $\sin\left(\frac{2\pi}{3}\right) =$ (*ii*) $\sin\left(\frac{-4\pi}{3}\right) =$

(*iv*) $\sin^{-1}\left(-\frac{\sqrt{3}}{2}\right) =$

- (b) Evaluate the following:

(*i*) $\sin^{-1}(-1) =$ (*ii*) $\cos^{-1}\left(-\frac{1}{\sqrt{2}}\right) =$ (*ii*) $\tan^{-1}\left(\frac{-\sqrt{3}}{3}\right) =$

(*iv*) $\csc^{-1}\left(-\frac{2}{\sqrt{3}}\right) =$ (*v*) $\sec^{-1}(2) =$ (*vi*) $\cot^{-1}\left(\frac{-\sqrt{3}}{1}\right) =$

(D) Solving Trig Expressions

13. Evaluate the following:

a) $\sin\left(\sin^{-1}\frac{24}{25}\right)$ b) $\sin\left(\cos^{-1}\frac{24}{25}\right)$ c) $\sin\left(\sin^{-1}\frac{25}{24}\right)$

14. Evaluate the following:

a) $\sin^{-1}\left[\sin\left(-\frac{\pi}{4}\right)\right]$ b) $\tan^{-1}\left(\tan\frac{5\pi}{4}\right)$ c) $\cos^{-1}\left(\sin\frac{5\pi}{3}\right)$

(D) Solving Trig Equations

- (c) Evaluate the following:

(*i*) $\cos(\sin^{-1}(-1)) =$ (*ii*) $\tan\left(\cos^{-1}\left(-\frac{1}{\sqrt{2}}\right)\right) =$ (*ii*) $\sec\left(\tan^{-1}\left(\frac{-\sqrt{3}}{3}\right)\right) =$

(*iv*) $\csc^{-1}\left(\sin\left(\frac{\pi}{4}\right)\right) =$ (*v*) $\sec^{-1}\left(\cos\left(\frac{2\pi}{3}\right)\right) =$ (*vi*) $\cot^{-1}(\tan(\pi)) =$

Homework

- HW
- S13.6, p871-872, Q3,13-43eol,51