Lesson 41 – Angles in Standard Position & Radian Measure

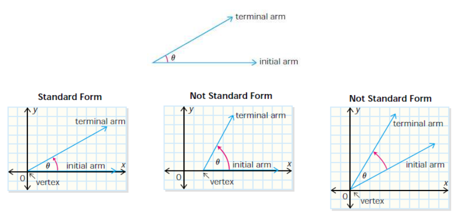
Math 2 Honors - Santowski

Fast Five

- Evaluate sin(50°) ➔ illustrate with a diagram
- Evaluate sin(130°) ➔ illustrate with a diagram
- Evaluate sin(230°) ➔ illustrate with a diagram??
- Evaluate sin(320°) ➔ illustrate with a diagram??
- Evaluate sin(770°) ➔ illustrate with a diagram??
- Evaluate sin(-50°) ➔ illustrate with a diagram??
- Use your calculator and graph the function f(x) = sin(x) on the domain -720°< x < 720°

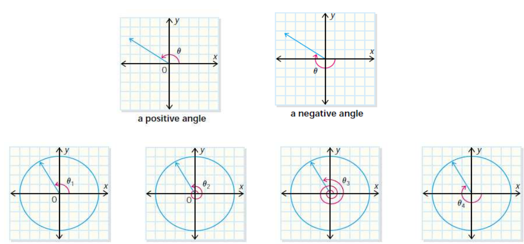
(A) Angles in Standard Position

- Angles in standard position are defined as angles drawn in the Cartesian plane where the initial arm of the angle is on the *x* axis, the vertex is on the origin and the terminal arm is somewhere in one of the four quadrants on the Cartesian plane



(A) Angles in Standard Position

- To form angles of various measure, the terminal arm is simply rotated through a given angle

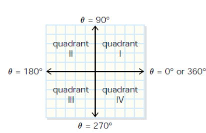


(A) Angles in Standard Position

- We will divide our Cartesian plane into 4 quadrants, each of which are a multiple of 90 degree angles

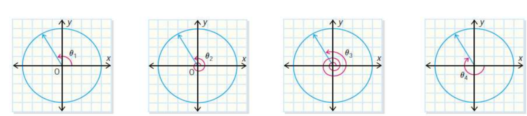
The *x-y* plane is divided into four **quadrants** by the *x*- and *y*-axes. If θ is a positive angle, then the terminal arm lies in

- quadrant I when $0° < \theta < 90°$
- quadrant II when $90° < \theta < 180°$
- quadrant III when $180° < \theta < 270°$
- quadrant IV when $270° < \theta < 360°$



(A) Coterminal Angles

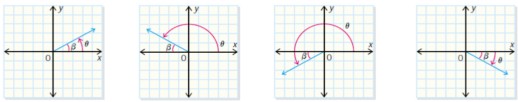
- **Coterminal angles** share the same terminal arm and the same initial arm.
- As an example, here are four different angles with the same terminal arm and the same initial arm.



If $\theta_1 = 120°$, then $\theta_2 = 360° + 120° = 480°$, $\theta_3 = 720° + 120° = 840°$, $\theta_4 = -360° + 120° = -240°$

(A) Principle Angles and Related Acute Angles

- The **principal angle** is the angle between 0° and 360°.
- The coterminal angles of 480°, 840°, and 240° all sha re the same principal angle of 120°.
- The **related acute angle** is the angle formed by the terminal arm of an angle in standard position and the *x*-axis.
- The related acute angle is always positive and lies between 0° and 90°.



(B) Examples

- **Example 1**

  Determine the principal angle and the related acute angle for $\theta = -225°$.

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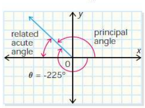
Solution

Sketch $\theta = -225°$ terminating in quadrant II. Label the principal angle and the related acute angle.



The principal angle is the smallest positive angle that is coterminal to $-225°$. In this case, $360° - 225° = 135°$. The related acute angle lies between the terminal arm and the *x*-axis. It is positive but less than 90°. In this case, $|-225° - (-180°)| = 45°$. Or, using the principal angle, $180° - 135° = 45°$.

(B) Examples

- **Example 2**

  Determine the next two consecutive positive coterminal angles and the first negative coterminal angle for 43°.

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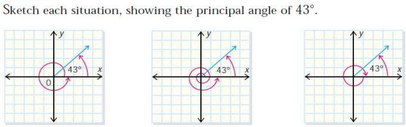
Solution

Sketch each situation, showing the principal angle of 43°.



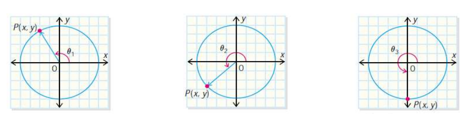
(a) The first positive coterminal angle for 43° is $360° + 43° = 403°$.

(b) The second coterminal angle is $360° + 360° + 43° = 763°$.

(c) The first negative coterminal angle is $-360° + 43° = -317°$.

(C) Ordered Pairs & Right Triangle Trig

- To help discuss angles in a Cartesian plane, we will now introduce ordered pairs to place on the terminal arm of an angle



$90° < \theta_1 < 180°$ — θ_1 terminates in quadrant II.

$180° < \theta_2 < 270°$ — θ_2 terminates in quadrant III.

$P(x, y)$ lies in the negative y-axis. $\theta_3 = 270°$

(C) Ordered Pairs & Right Triangle Trig

- Point *P* (3, 4) is on the terminal arm of an angle, θ, in standard position.
- **(a)** Sketch the principal angle, θ.
- **(b)** Determine the sine, cosine & tangent ratios of θ.
- **(c)** Determine the value of the related acute angle to the nearest degree.
- **(d)** What is the measure of θ to the nearest degree?

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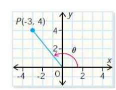
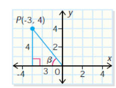




(C) Ordered Pairs & Right Triangle Trig - Examples

- Point $P(-9, 4)$ is on the terminal arm of an angle in standard position.
  - (a) Sketch the principal angle, θ.
  - (b) What is the measure of the related acute angle to the nearest degree?
  - (c) What is the measure of θ to the nearest degree?

Point $P(-5, -3)$ is on the terminal arm of an angle, θ, in standard position.
(a) Sketch the principal angle, θ.
(b) What is the measure of the related acute angle to the nearest degree?
(c) What is the measure of θ to the nearest degree?
(d) What is the measure of the first negative coterminal angle?

Point $P(-5, -8)$ is on the terminal arm of an angle, θ, in standard position. Determine all values of θ for $-540° \leq \theta \leq 270°$.

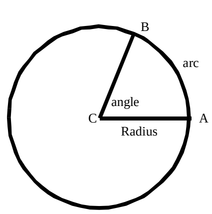
(B) Radians

- We can measure angles in several ways - one of which is degrees
- Another way to measure an angle is by means of radians
- One definition to start with ➔ an arc is a distance along the curve of the circle ➔ that is, part of the circumference
- One radian is defined as the measure of the angle subtended at the center of a circle by an arc equal in length to the radius of the circle
- Now, what does this mean?

(B) Radians

If we rotate a terminal arm (BC) around a given angle, then the end of the arm (at point B) moves along the circumference from A to B

If the distance point B moves is equal in measure to the radius, then the angle that the terminal arm has rotated is defined as one radian



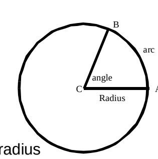
If B moves along the circumference a distance twice that of the radius, then the angle subtended by the arc is 2 radians

So we come up with a formula of θ = arc length/radius = s/r

(C) Converting between Degrees and Radians

If point B moves around the entire circle, it has revolved or rotated 360°

Likewise, how far has the tip of the terminal arm traveled? One circumference or $2\pi r$ units.



So in terms of radians, the formula is θ = arc length/radius
$\theta = s/r = 2\pi r/r = 2\pi$ radians

So then an angle of $360° = 2\pi$ radians or more easily, an angle of $180° = \pi$ radians

(C) Converting from Degrees to Radians

- Our standard set of first quadrant angles include 0°, 30°, 45°, 60°, 90° and we now convert them to radians:
- We can set up equivalent ratios as:
- 30° =
- 45° =
- 60° =
- 90° =

(C) Converting from Degrees to Radians

- Our standard set of first quadrant angles include 0°, 30°, 45°, 60°, 90° and we now convert them to radians:
- We can set up equivalent ratios as:
- 30°/x radians = 180°/π radians
- Then x = π/6 radians
- 45°/x = 180°/π ➔ x = π/4 radians
- 60°/x = 180°/π ➔ x = π/3 radians
- 90°/x = 180°/π ➔ x = π/2 radians

(D) Converting from Radians to Degrees

- Let's work with our second quadrant angles with our equivalent ratios:
- 2π/3 radians
- 3π/4 radians
- 5π/6 radians

(D) Converting from Radians to Degrees

- Let's work with our second quadrant angles with our equivalent ratios:
- 180°/π = x / (2π/3)
  - ➔ x = (2π/3)(180/π) = 120°
- 180°/π = x / (3π/4)
  - ➔ x = (3π/4)(180/π) = 135°
- 180°/π = x / (5π/6)
  - ➔ x = (5π/6)(180/π) = 150°

(E) Table of Equivalent Angles

- We can compare the measures of important angles in both units on the following table:

0 °	90°	180°	270°	360°

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- We can compare the measures of important angles in both units on the following table:

0 °	90°	180°	270°	360°
0 rad	π/2 rad	π rad	3π/2 rad	2π rad

(E) Table of Equivalent Angles

- We can compare the measures of important angles in both units on the following table:

30	45	60	120	135	150	210	225	240	300	315	330
$\pi/6$	$\pi/4$	$\pi/3$	$2\ \pi/3$	$3\pi/4$	$5\pi/6$	$7\pi/6$	$5\pi/4$	$4\pi/3$	$5\pi/3$	$7\pi/4$	$11\pi/6$

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(H) Internet Links

- Topics in trigonometry: Radian measure from The Math Page
- Measurement of angles from David Joyce, Clark University
- Radians and Degrees - on-line Math Lesson from TV

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(I) Homework

- Angles in Standard Position: Nelson text, p442, Q2,3,5,6 (on-line link)

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