Lesson 40 – Sine Law & The Ambiguous Case

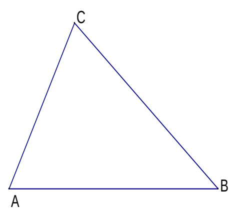
Math 2 Honors - Santowski

Lesson Objectives

- Understand from a geometric perspective WHY the ambiguous case exists
- Understand how to identify algebraically that their will be 2 solutions to a given sine law question
- Solve the 2 triangles in the ambiguous case
- See that the sine ratio of a acute angle is equivalent to the sine ratio of its supplement

(A) Review of the Sine Law

- If we have a non right triangle, we cannot use the primary trig ratios, so we must explore new trigonometric relationships.
- One such relationship is called the Sine Law which states the following:

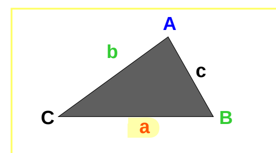


$$\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C} \quad \text{OR} \quad \frac{\sin A}{a} = \frac{\sin B}{b} = \frac{\sin C}{c}$$

Law of Sines: Solve for Sides

Have: two angles, one side opposite one of the given angles

Solve for: missing side opposite the other given angle



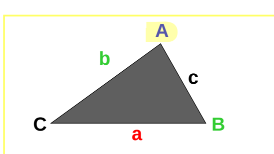
Missing Side

$$\frac{a}{\sin A} = \frac{b}{\sin B}$$

Law of Sines: Solve for Angles

Have: two sides and one of the opposite angles

Solve for: missing angle opposite the other given angle



Missing Angle

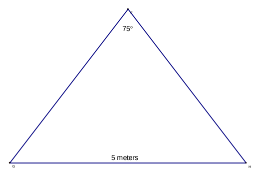
$$\frac{a}{\sin A} = \frac{b}{\sin B}$$

(B) Examples Sine Law

- We can use these new trigonometric relationships in solving for unknown sides and angles in acute triangles:
- ex 4. Find A in ABC if a = 10.4, c = 12.8 and C = 75°
- ex 5. Find a in ABC if A = 84°, B = 36°, and b = 3.9
- ex 6. Solve EFG if E = 82°, e = 11.8, and F = 25°
- There is one limitation on the Sine Law, in that it can only be applied if a side and its opposite angle is known. If not, the Sine Law cannot be used.

(B) Examples Sine Law

- Mark is a landscaper who is creating a triangular planting garden. The homeowner wants the garden to have two equal sides and contain an angle of 75°. Also, the longest side of the garden must be exactly 5 m.
  - **(a)** How long is the plastic edging that Mark needs to surround the garden?
  - **(b)** Determine the area of the garden.

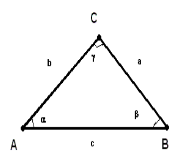


(C) Inverse Trig – Input/Output

- Determine the measure of an angle whose sine ratio is 0.75
- Solve the equation sin(*x*) = 0.75 for *x*
- Solve the equation $x = \sin^{-1}(0.75)$
- What is the difference in meaning amongst these 3 questions??
- Solve the following equations for x:
- $x = \sin^{-1}(0.89)$ $x = \cos^{-1}(.11)$
- $x = \sin^{-1}(0.25)$ $x = \tan^{-1}(3.25)$
- sin(x) = 0.45 sin(x) = 0.6787
- Explain why it is IMPOSSIBLE to solve $\sin^{-1}(1.25) = x$

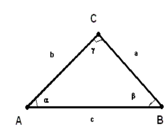
(D) Sine Law – Scenario #1

- Let's work through 2 scenarios of solving for ∠B :
- Let ∠A = 30°, a = 3 and b = 2 (so the longer of the two given sides is opposite the given angle)
- Then sin β = b sin α / a
- And sin β = 2 sin 30 / 3
- So ∠B = 19.5°



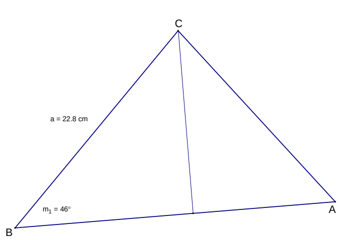
(D) Sine Law – Scenario #2

- In our second look, let's change the measures of a and b, so that a = 2 and b = 3 (so now the shorter of the two given sides is opposite the given angle)
- Then sin β = b sin α / a
- And sin β = 3 sin 30 / 2
- So ∠B = 48.6°
- BUT!!!!! …….. there is a minor problem here …..

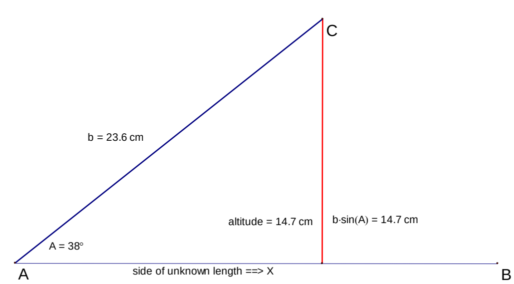


(E) Altitude (height)

- Explain how to find the height (altitude) of this non-right triangle



(E) Altitude (height)

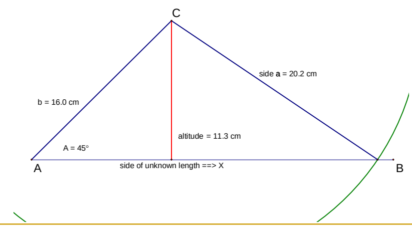


(F) Considerations with Sine Law

- If you are given information about non-right triangle and you know 2 angles and 1 side, then ONLY one triangle is possible and we never worry in these cases
- If you know 2 sides and 1 angle, then we have to consider this “ambiguous” case issue
  - If the side opposite the given angle IS THE LARGER of the 2 sides ➔ NO WORRIES
  - **If the side opposite the given angle IS THE SHORTER of the 2 sides ➔ ONLY NOW WILL WE CONSIDER THIS “ambiguous” case**
- WHY????

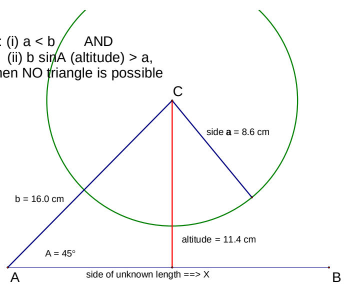
Case #1 – if a>b

If: (i) a > b, then ONE OBTUSE triangle is possible



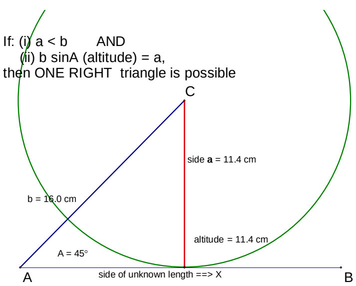
Case #2

If: (i) a < b AND
(ii) b sinA (altitude) > a,
then NO triangle is possible



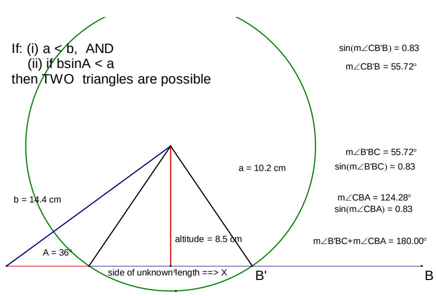
Case #3

If: (i) a < b AND
(ii) b sinA (altitude) = a,
then ONE RIGHT triangle is possible



Case #4 – the Ambiguous Case

If: (i) a < b, AND
(ii) if bsinA < a
then TWO triangles are possible



Case #4 – the Ambiguous Case

If: (i) a < b, AND
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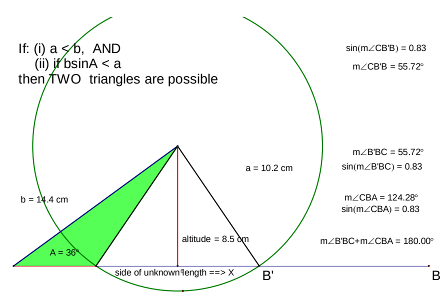
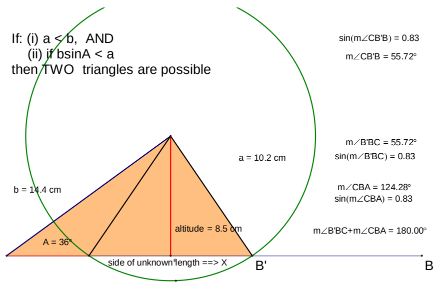
then TWO triangles are possible



Case #4 – the Ambiguous Case



Summary

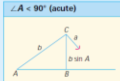
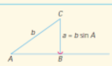
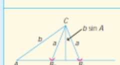
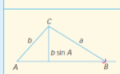
- Case 1 ➔ if we are given 2 angles and one side ➔ proceed using sine law
- Case 2 ➔ if we are given 1 angle and 2 sides and the side opposite the given angle is LONGER ➔ proceed using sine law

- if we are given 1 angle and 2 sides and the side opposite the given angle is SHORTER ➔ proceed with the following "check list"

- Case 3 ➔if the product of "bsinA > a", NO triangle possible
- Case 4 ➔ if the product of "bsinA = a", ONE triangle
- Case 5 ➔ if the product of "bsinA < a" TWO triangles

- RECALL that "bsinA" represents the altitude of the triangle

Summary

∠A < 90° (acute)	Conditions	Number and Type of Triangles Possible
	$a < b \sin A$	no triangle
	$a = b \sin A$	one right triangle
	$b \sin A < a < b$	two triangles—one acute, one obtuse
	$a \geq b$	one triangle

Examples of Sine Law

- if ∠ A = 44º and ∠ B = 65º and b=7.7 find the missing information.

Examples of Sine Law

- if ∠ A =44.3º and a=11.5 and b=7.7 find the missing information.

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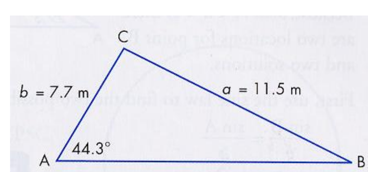
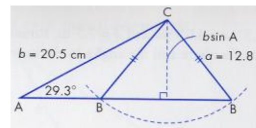


Examples of Sine Law

- if ∠ A=29.3º and a=12.8 and b = 20.5

Examples of Sine Law

- if ∠ A=29.3 and a=12.8 and b = 20.5
- All the other cases fail, because bsinA<a<b 10<a (12.8)<20.5, which is true.
- Then we have two triangles, solve for both angles



Examples of Sine Law

- Solve triangle *PQR* in which ∠ *P* = 63.5° and ∠ *Q* = 51.2° and *r* = 6.3 cm.

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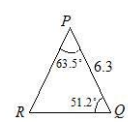

Examples of Sine Law

- ex. 1. In ΔABC, ∠ A = 42°, a = 10.2 cm and b = 8.5 cm, find the other angles
- ex. 2. Solve ΔABC if ∠ A = 37.7, a = 30 cm, b = 42 cm

Examples of Sine Law

- ex. 1. In Δ ABC, ∠ A = 42°, a = 10.2 cm and b = 8.5 cm, find the other angles
- First test ➔side opposite the given angle is longer, so no need to consider the ambiguous case ➔ i.e. a > b ➔ therefore only one solution
- ex. 2. Solve Δ ABC if ∠ A = 37.7, a = 30 cm, b = 42 cm
- First test ➔ side opposite the given angle is shorter, so we need to consider the possibility of the "ambiguous case" ➔ a < b ➔ so there are either 0,1,2 possibilities.
- So second test is a calculation ➔ Here a (30) > b sin A (25.66), so there are two cases

Homework

- HW

- Nelson Questions: any of Q5,6,8