

## Lesson 39 - Review of Right Triangle Trigonometry

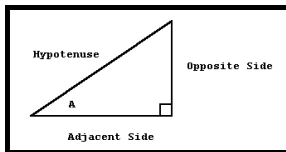
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### (A) Review of Right Triangle Trig

Trigonometry is the study and solution of Triangles. Solving a triangle means finding the value of each of its sides and angles. The following terminology and tactics will be important in the solving of triangles.

- Pythagorean Theorem ( $a^2+b^2=c^2$ ). Only for right angle triangles
- Sine (sin), Cosecant (csc or  $1/\sin$ )
- Cosine (cos), Secant (sec or  $1/\cos$ )
- Tangent (tan), Cotangent (cot or  $1/\tan$ )
- Right/Oblique triangle

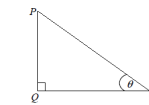
### (A) Review of Right Triangle Trig



- In a right triangle, the primary trigonometric ratios (which relate pairs of sides in a ratio to a given reference angle) are as follows:
- sine A = opposite side/hypotenuse side
- cosine A = adjacent side/hypotenuse side
- tangent A = adjacent side/opposite side
- recall SOHCAHTOA as a way of remembering the trig. ratio and its corresponding sides

### (A) Review of Right Triangle Trig

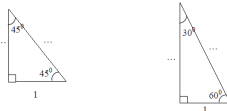
#### ✂ Trigonometric Ratios of Acute Angles



$\cos \theta = \frac{\dots}{\dots}$	;	$\sec \theta = \frac{\dots}{\dots}$
$\sin \theta = \frac{\dots}{\dots}$	;	$\csc \theta = \frac{\dots}{\dots}$
$\tan \theta = \frac{\dots}{\dots}$	;	$\cot \theta = \frac{\dots}{\dots}$

### (A) Review of Right Triangle Trig

#### ✂ Trigonometric Ratios of Some Special Angles

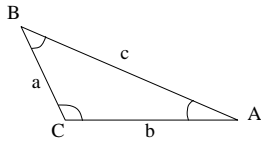


Angle	sin	cos	tan	csc	sec	cot
$0^\circ$						
$30^\circ$						
$45^\circ$						
$60^\circ$						
$90^\circ$						

### (B) Examples – Right Triangle Trigonometry

- Using the right triangle trig ratios, we can solve for unknown sides and angles:
- ex 1. Find  $a$  in ABC if  $b = 2.8$ ,  $C = 90^\circ$ , and  $A = 35^\circ$
- ex 2. Find  $A$  in ABC if  $c = 4.5$  and  $a = 3.5$  and  $B = 90^\circ$
- ex 3. Solve ABC if  $b = 4$ ,  $a = 1.5$  and  $B = 90^\circ$

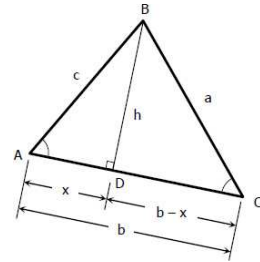
### (C) Cosine Law



- The Cosine Law states the following:
  - $a^2 = b^2 + c^2 - 2bc\cos A$
  - $b^2 = a^2 + c^2 - 2accosB$
  - $c^2 = a^2 + b^2 - 2abcosC$
- We can use the Cosine Law to work in right and non-right triangles (oblique) in which we know all three sides (SSS) and one in which we know two sides plus the contained angle (SAS).

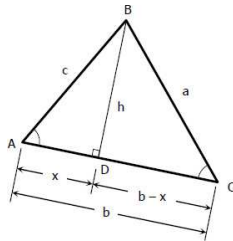
### (D) Cosine Law - Derivation

- Cosine function for triangle ADB**  
 $\cos A = c/x \rightarrow x = c\cos A$
- Pythagorean theorem for triangle ADB**  
 $x^2 + h^2 = c^2 \rightarrow h^2 = c^2 - x^2$
- Pythagorean theorem for triangle CDB**  
 $(b-x)^2 + h^2 = a^2$



### (D) Cosine Law - Derivation

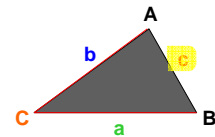
- Pythagorean theorem for triangle CDB**  
 $(b-x)^2 + h^2 = a^2$
- Substitute  $h^2 = c^2 - x^2$**   
 $(b-x)^2 + (c^2 - x^2) = a^2$   
 $(b^2 - 2bx + x^2) + (c^2 - x^2) = a^2$   
 $b^2 - 2bx + c^2 = a^2$
- Substitute  $x = c \cos A$**   
 $b^2 - 2b(c\cos A) + c^2 = a^2$
- Rearrange:**  
 $a^2 = b^2 + c^2 - 2bc\cos A$



### (E) Law of Cosines:

Have: two sides, included angle

Solve for: **missing side**



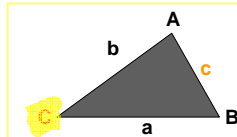
$$c^2 = a^2 + b^2 - 2ab \cos C$$

(missing side)<sup>2</sup> = (one side)<sup>2</sup> + (other side)<sup>2</sup> - 2(one side)(other side) cos(included angle)

### (E) Law of Cosines:

Have: three sides

Solve for: **missing angle**

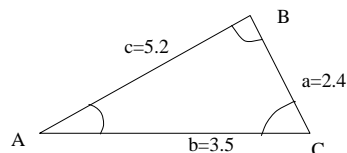


$$\cos C = \frac{a^2 + b^2 - c^2}{2ab}$$

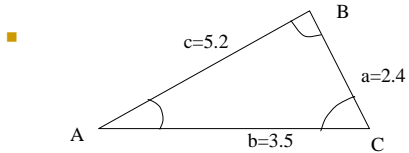
Missing Angle  $\rightarrow$   $\cos C$   $\leftarrow$  Side Opposite Missing Angle

### (F) Cosine Law - Examples

- Solve this triangle



## (F) Cosine Law - Examples



Start with the law of cosines because there are no angles given.  
 $a^2 = b^2 + c^2 - 2bc \cos A$ . Substitute values.  $2.4^2 = 3.5^2 + 5.2^2 - 2(3.5)(5.2) \cos A$ ,  
 $5.76 - 12.25 - 27.04 = -2(3.5)(5.2) \cos A$ ,  $33.53 = 36.4 \cos A$ ,  $33.53/36.4 = \cos A$ ,  $0.921 = \cos A$ ,  $A = 67.07$ .  
Now for B.  
 $b^2 = a^2 + c^2 - 2ac \cos B$ ,  $(3.5)^2 = (2.4)^2 + (5.2)^2 - 2(2.4)(5.2) \cos B$ ,  $12.25 = 5.76 + 27.04 - 24.96 \cos B$ ,  
 $12.25 = 5.76 + 27.04 - 24.96 \cos B$ ,  $12.25 - 5.76 - 27.04 = -24.96 \cos B$ ,  $20.54/24.96 = \cos B$ ,  $0.823 = \cos B$ ,  
 $B = 34.61$ .  
 $C = 180 - 34.61 - 67.07 = 78.32$ .

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## (F) Examples Cosine Law

- We can use these new trigonometric relationships in solving for unknown sides and angles in acute triangles:
- ex 7. Find  $c$  in CDE if  $C = 56^\circ$ ,  $d = 4.7$  and  $e = 8.5$
- ex 8. Find  $G$  in GHJ if  $h = 5.9$ ,  $g = 9.2$  and  $j = 8.1$
- ex 9. Solve CDE if  $D = 49^\circ$ ,  $e = 3.7$  and  $c = 5.1$

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## (G) Links

- For help with right triangle trig:  
<http://id.mind.net/~zona/mmts/trigonometryRealms/introduction/rightTriangle/trigRightTriangle.html>
- For help with the Sine Law  
<http://www.themathpage.com/aTrig/law-of-sines.htm>
- For help with the Cosine Law  
<http://www.themathpage.com/aTrig/law-of-cosines.htm>

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## (H) Homework

- p. 833, Q# 21, 22, 25, 26, 29, 30, 33-43 odds (RTT)
- P. 899, Q# 5,6,13-18, 46-50 (LoC)

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