Lesson 36 – Logarithmic Models

Math 2 Honors - Santowski

(A) Introduction

- What do we USE logarithms for???
- We will see 3 types of applications of logarithms ….

(A) Introduction

- Many measurement scales used for naturally occurring events like earthquakes, sound intensity, and acidity make use of logarithms
- We will now consider several of these applications, having our log skills in place

(B) Earthquakes and The Richter Scales

- For studying earthquakes, we have a log based function:
- ➔ ***R*** = log(***a***/***T***) + ***B***, where ***R*** is the Richter scale magnitude, ***a*** is the amplitude of the vertical ground motion (measured in microns), ***T*** is the period of the seismic wave (measured in seconds) and ***B*** is a factor that accounts for the weakening of the seismic waves
- So, determine the intensity of an earthquake if the amplitude of vertical ground motion is 150 microns, the period of the wave is 2.4 s, and B = 2.4

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- So, determine the intensity of an earthquake if the amplitude of vertical ground motion is 150 microns, the period of the wave is 2.4 s, and B = 2.4
- R = log(150/2.4) + 2.4
- R = log(62.5) + 2.4
- R = 4.2

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- 5.5 = log(***a***/1.8) + 3.2
- 2.3 = log(***a***/1.8)
- $10^{(2.3)}$ = ***a***/1.8
- $1.8(10^{(2.3)})$ = a
- 359.1 microns = a

(B) Earthquakes and The Richter Scales

- Another formula for comparison of earthquakes uses the following formula ➔ we can compare intensities of earthquakes using the formula:
- $\log(I_1/I_2) = \log(I_1/S) - \log(I_2/S)$ where I_1 is the intensity of the more intense earthquake and I_2 is the intensity of the less intense earthquake and $\log(I_1/S)$ refers to the magnitude of a given earthquake.
- ex. The recent Haiti earthquake had a magnitude of 7.0 on the Richter scale while a moderately destructive earthquake has a magnitude of 5.75. How many times more intense was the Haiti earthquake?
- ex. The San Francisco earthquake of 1906 had a magnitude of 8.3 on the Richter scale while an earthquake of magnitude 5.0 can be felt, but is rarely destructive. How many times more intense was the San Francisco earthquake?

(C) Sound Intensity

- Loudness of sounds is measured in decibels. The loudness of a sound is always given in reference to a sound at the threshold of hearing (which is assigned a value of 0 dB.)
- The formula used to compare sounds is $y = 10 \log (i/i_r)$ where *i* is the intensity of the sound being measured, i_r is the reference intensity and *y* is the loudness in decibels.
- ex. If a sound is 100 times more intense than the threshold reference, then the loudness of this sound is...?
- ex. Your defective muffler creates a sound of loudness 125 dB while my muffler creates a sound of 62.5 dB. How many times more intense is your muffler than mine?

(D) Scales of Acidity - pH

- the pH scale is another logarithmic scale used to measure the acidity or alkalinity of solutions
- a neutral pH of 7 is neither acidic nor basic and acidic solutions have pHs below 7, while alkaline solutions have pHs above 7
- Mathematically, pH = -log (concentration of H^+) ➔ so the concentration of H^+ in a neutral solution is 1×10^{-7} moles/L
- an increase in 1 unit on the pH scale corresponds to a 10 fold decrease in acidity (for acidic solutions) while an increase in 1 pH unit for bases corresponds to a 10 fold increase in alkalinity
- ex 3. If the pH of apple juice is 3.1 and the pH of milk is 6.5, how many more times acidic is apple juice than milk?

(E) Changing Bases

- Is 4 a power of 2?
- Is 8 a power of 2?
- Is 1024 a power of 2?
- What about 7? Is 7 a power of 2??

(E) Changing Bases

- What about 7? Is 7 a power of 2??
- Well, as an equation, we would write it as ??
- $2^x = 7$ ➔ its easy to solve graphically, nut what about about algebraically?
- Let's use our "common" base of 10 to rewrite each base
- $2 = 10^a$ and likewise $7 = 10^b$
- So we choose to rewrite our original question using a "common" base as ➔ $(10^a)^x = (10^b)$
- Which suggests that the exponents are equal, hence ax = b and hence x = b/a

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- But how does that help?
- if $2 = 10^a$ ➔ then $a = \log_{10}(2)$, likewise $b = \log_{10}(7)$
- So if x = b/a ➔ then x = log7/log2

(E) Changing Bases

- if $2 = 10^a$ ➔ then $a = \log_{10}(2)$, likewise $b = \log_{10}(7)$
- So if x = b/a ➔ then x = log7/log2 = 2.807
- So going back to the original equation ($2^x = 7$) ➔ we have $2^{(2.807)} = 7$
- So could we develop/predict a general formula that will allow us to change from one base (7) to another base (2)?

(E) Changing Bases

- So in general, if I want to change from base ***b*** to base ***a***, I would solve the equation $a^x = b$
- The solution would be ……

(E) Changing Bases

- So in general, if I want to change from base ***b*** to base ***a***, I would solve the equation $a^x = b$
- The solution would be …… x = logb/loga
- So $a^{\frac{\log b}{\log a}} = b$

(E) Creating Exponential & Logarithmic Models – Linearizing Data

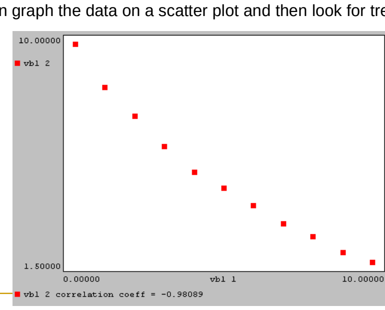
- We can analyze data gathered from some form of "experiment" and then use our math skills to develop equations to summarize the information:
- Consider the following data of drug levels in a patient:

Time	0	1	2	3	4	5	6	7	8	9	10
Drug level	10	8.3	7.2	6.0	5.0	4.4	3.7	3.0	2.5	1.9	1.5

- Create an algebraic model to describe the data

(E) Creating Exponential & Logarithmic Models

- We can graph the data on a scatter plot and then look for trends:

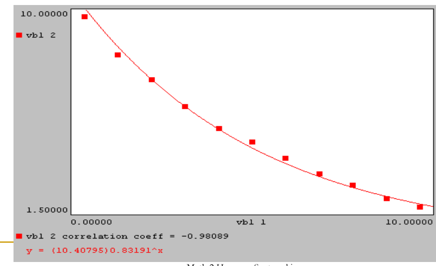


(E) Creating Exponential & Logarithmic Models

- We may suspect the data to be exponential/geometric, so we could look for an average common ratio (y_2/y_1) ➔ which we can set up easily on a spreadsheet and come up with an average common ratio of 0.8279
- So a geometric formula could be $N(t) = N_0(r)^t$ so we could propose an equation like $N(t) = 10(0.8279)^t$
- We could use graphing software to generate the equation for us as:

(E) Creating Exponential & Logarithmic Models

- We could use graphing software to generate the equation for us as:
- $N(t) = 10.41(0.8318)^t$

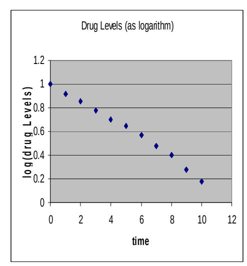


(E) Creating Exponential & Logarithmic Models

- Or we can make use of logarithms and manipulate the data so that we generate a linear graph ➔ we do this by taking the logarithm of our drug level values and then graphing time vs the logarithm of our drug levels
- This data can be presented and displayed as follows:

(E) Creating Exponential & Logarithmic Models

Time	Drug Levels (as logarithm)
0	1
1	0.919078
2	0.857332
3	0.778151
4	0.69897
5	0.643453
6	0.568202
7	0.477121
8	0.39794
9	0.278754



(E) Creating Exponential & Logarithmic Models

- Determine the equation of this line $y = mx + b$
- ➔ $y = -0.07992x + 1.0174$ with $r = -0.9964$

- Now we need to "readjust" the equation:
- $\log(\text{drug level}) = -0.07992(t) + 1.0174$
- $\log_{10}(N) = -0.07992(t) + 1.0174$
- $10^{(-0.07992t + 1.0174)} = N$
- $[10^{(-0.07992t)}] \times [10^{(1.0174)}] = N$

- $10.41(0.8319^t) = N(t)$
- Which is very similar to the equation generated in 2 other ways (common ratio & GDC)

(F) Internet Links

- You can try some on-line word problems from U of Sask EMR problems and worked solutions

- More work sheets from EdHelper's Applications of Logarithms: Worksheets and Word Problems

(E) Homework

- p. 389 # 15-27 odds, 37-41 odds, 45-47, 55, 57
- Additional Problems from Nelson Text (scanned and attached on website)
- P140-2, Q3,4,5,7,8,12,15