Lesson 34– The Logarithmic Function – An Inverse Perspective

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(A) Graph of Exponential Functions

- Graph the exponential function $f(x) = 2^x$ by making a table of values
- What does the input/domain of the function represent?
- What does the output/range of the function represent?
- What are the key graphical features of the function $f(x) = 2^x$?

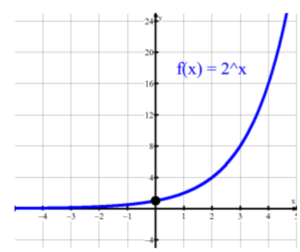
(A) Table of Values for Exponential Functions

x	y
-5.00000	0.03125
-4.00000	0.06250
-3.00000	0.12500
-2.00000	0.25000
-1.00000	0.50000
0.00000	1.00000
1.00000	2.00000
2.00000	4.00000
3.00000	8.00000
4.00000	16.00000
5.00000	32.00000
6.00000	64.00000
7.00000	128.00000

- what does $f(-2) = ¼$ mean ➔ $2^{-2} = ¼$
- Domain/input ➔ the value of the exponent to which the base is raised
- Range/output ➔ the result of raising the base to the specific exponent (i.e. the power)
- Graphical features ➔ y-intercept (0,1); asymptote; curve always increases

(A) Graph of Exponential Functions

x	y
-5.00000	0.03125
-4.00000	0.06250
-3.00000	0.12500
-2.00000	0.25000
-1.00000	0.50000
0.00000	1.00000
1.00000	2.00000
2.00000	4.00000
3.00000	8.00000
4.00000	16.00000
5.00000	32.00000
6.00000	64.00000
7.00000	128.00000



(B) Inverse of Exponential Functions

- List the ordered pairs of the inverse function and then graph the inverse
- Let's call the inverse I(x) for now ➔ so what does I(¼) mean and equal ➔ I(¼) = -2
- of course $I(x) = f^{-1}(x)$, so what am I asking if I write $f^{-1}(¼)$ = ????
- After seeing the graph, we can analyze the features of the graph of the logarithmic function

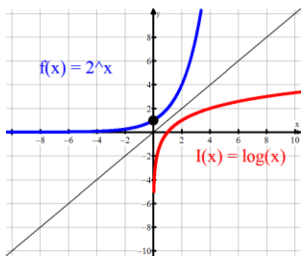
(B) Table of Values for the Inverse Function

x	y
0.03125	-5.00000
0.06250	-4.00000
0.12500	-3.00000
0.25000	-2.00000
0.50000	-1.00000
1.00000	0.00000
2.00000	1.00000
4.00000	2.00000
8.00000	3.00000
16.00000	4.00000
32.00000	5.00000
64.00000	6.00000
128.00000	7.00000

- what does $f^{-1}(¼) = -2$ mean
  - ➔ $2^{-2} = ¼$
- Domain/input ➔ the power
- Range/output ➔ the value of the exponent to which the base is raised that produced the power
- Graphical features ➔ x-intercept (1,0); asymptote; curve always increases

(B) Table of Values & Graphs for the Inverse Function

- x y
- 0.03125 -5.00000
- 0.06250 -4.00000
- 0.12500 -3.00000
- 0.25000 -2.00000
- 0.50000 -1.00000
- 1.00000 0.00000
- 2.00000 1.00000
- 4.00000 2.00000
- 8.00000 3.00000
- 16.00000 4.00000
- 32.00000 5.00000
- 64.00000 6.00000
- 128.00000 7.00000



(C) TheLogarithmic Function

- If the inverse is $f^{-1}(x)$ so what I am really asking for if I write $f^{-1}(1/32) = -5$
- The equation of the inverse can be written as $x = 2^y$.
- But we would like to write the equation EXPLICITLY (having the y isolated ➔ having the exponent isolated)
- This inverse is called a logarithm and is written as $y = \log_2(x)$

(C) Terminology (to clarify … I hope)

- In algebra, the terms EXPONENT and POWER unfortunately are used interchangeably, leading to confusion.
- We will exclusively refer to the number that the base is raised to AS THE EXPONENT and NOT THE POWER.
- For the statement that $2^3 = 8$,
  - a) the base is 2: the base is the number that is repeatedly multiplied by itself.
  - b) the exponent is 3: the exponent is the number of times that the base is multiplied by itself.
  - c) the power is 8: the power is the ANSWER of the base raised to an exponent, or the product of repeatedly multiplying the base by itself an exponent number of times.

(C) Terminology (to clarify … I hope)

- Other ways to read $2^3 = 8$:
- ☐ "Two cubed is 8."
- ☐ "Two to the exponent 3 is 8."
- ☐ "Two to the 3 is 8."
- ☐ "Eight is the third power of 2."
- ☐ (Eight is the product of two multiplied by itself 3 times.)
- Why is "two to the third" incorrect?
- Why is "two to the power three" incorrect?

(C) Terminology (to clarify … I hope)

- Identify the parts of an exponential equation. State the base, the exponent and the power for each.

	$b^x=p$	$(-4)^3 = 64$	$2^{-3} = \frac{1}{8}$	$j^0 = 1$	$\sqrt[3]{n} = z$	$(\sqrt[3]{n})^k = y$
Base						
Exponent						
Power						

(C) Terminology (to clarify … I hope)

- So what's the deal with the terminology ???
- Given that $f(x) = 2^x$, I can write $f(5) = 2^5 = 32$ and what is meant is that the **EXPONENT 5** is "applied" to the **BASE 2**, resulting in 2 multiplied by itself 5 times (2x2x2x2x2) giving us the result of the **POWER of 32**
- The inverse of the exponential is now called a logarithm and is written as $y = \log_2(x)$ or in our case $5 = \log_2(32)$ and what is meant now is that I am taking the **logarithm of the POWER 32** (while working in **BASE 2**) and I get an **EXPONENT of 5** as a result!

(C) Terminology (to clarify … I hope)

- So the conclusion ➔ To PRODUCE the POWER, I take a BASE and exponentiate the base a given number of times (ie. The EXPONENT) ➔ this is the idea of an exponential function
- With a logarithmic function ➔ I start with a given POWER and determine the number of times the BASE was exponentiated (i.e. the EXPONENT)
- In math, we use shortcut notations ALL THE TIME. The mathematical shorthand for "What is the exponent on b (the base) that yields p (the power)?" is read as $\log_b p$.

Exponential versus Logarithmic Forms

- Exponential Form: $b^x = p$ Logarithmic Form: $\log_b p = x$
- 1. What 3 quantities are involved in the exponential form?
- 2. What quantity is isolated when using exponential form?
- 3. What quantity is the input when using the exponential form?
- 4. What 3 quantities are involved in the logarithmic form?
- 5. What quantity is isolated when using logarithmic form?
- 6. What quantity is the input when using the exponential form?

Exponential versus Logarithmic Forms

- Sometimes it is to our advantage to isolate other quantities, like the exponent.
- What is the exponent on (the base of) 2 to get (the power of):

a) 8	b) 16	c) 1/8	d) 1	e) 1024	f) 1/512	g) 0	h) -4

(D) Playing With Numbers

- Write the following equations in logarithmic form:
- a) $2^3 = 8$ b) $\sqrt{9} = 3$
- c) $125^{1/3} = 5$ d) $4^{-1/2} = ½$
- Write the following equations in exponential form:
- a) $\log_5 25 = 2$ b) $\log_2(1/16) = -4$
- c) $\log_4 2 = 0.5$ d) $\log_7 1 = 0$
- Evaluate the following:
- a) $\log_3(1/27)$ b) $\log_4 8$ c) $\log_{½} 4$ d) $\log_3 27 + \log_3 81$

(D) Common Logarithms

- A common logarithm is a logarithm that uses base 10. You can ignore writing the base in this case: $\log_{10} p = \log p$.
- Interpret and evaluate the following:
- a) log10 b) log100 c) log1000
- d) log1 e) log (1/10000) f) log $1/\sqrt{10}$
- Evaluate the following with your calculator and write the value to three decimal places).
- a) log9 b) log10 c) log11 d) $\log\pi$

(D) Working With Logarithms

- Solve: (HINT: switch to exponential form)
- (a) $\log_x 27 = 3$ (b) $\log_x \sqrt[3]{25} = 5$
- (c) $\log_x 8 = ¾$ (d) $\log_x 25 = 2/3$
- (e) $\log_4 \sqrt{2} = x$ (f) $\log_2 2^7 = x$
- (g) $5\log_3 9 = x$ (h) $\log_4 x = -3$
- (i) $\log_9 x = -1.5$ (j) $\log_2(x + 4) = 5$
- (k) $\log_3(x – 3) = 3$ (l) $\log_2(x^2 – x) = \log_2 12$
- (m) $\log_3 \sqrt[5]{9} = x$ (n) $\log_{1/3} 9\sqrt{27} = x$
- (o) $\log_x 81 = -4$ (p) $\log_2 \sqrt{0.125} = x$

(D) Working With Logarithms

- Evaluate $\log_3 9$ and $\log_9 3$
- Evaluate $\log_5 125$ and $\log_{125} 5$
- What relationship exists between the values of $\log_a b$ and $\log_b a$
- Solve the system defined by $2^{x+y} = 32$ and $2^{x-y} = 8$
- Solve $\log_7(\log_4 x) = 0$
- Solve $\log_5(\log_2(\log_3 x)) = 0$

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(E) Transformed Logarithmic Functions

- As will be seen in the next exercises, the graph maintains the same "shape" or characteristics when transformed
- Depending on the transformations, the various key features (domain, range, intercepts, asymptotes) will change

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(E) Transformed Logarithmic Functions

- So we can now do a complete graphic analysis of this graph
- (i) no y-intercept and the x-intercept is 1
- (ii) the y axis is an asymptote
- (iii) range {yER}
- (iv) domain $\{x > 0\}$
- (v) it increases over its domain
- (vi) it has no max/min or turning points

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(E) Graphing Log Functions

- Without using graphing technology, graph the following functions (it may help to recall your knowledge of function transformations)
- (1) $f(x) = \log_2(x + 2)$
- (2) $f(x) = -3\log_2(x - 4)$
- (3) $f(x) = \log_5(4x – 4) + 5$
- Examples and discussions on how to make these graphs is found at the following website:
- Graphs of Logarithmic Functions from AnalyzeMath

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(F) Homework

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