

Lesson 31 – Inequalities with Radical Equations

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Review - Solving Radical Equations - Examples

- Graphically solve $3 + \sqrt{x+1} = 2x$
- Algebraically solve $3 + \sqrt{x+1} = 2x$
- (domain for $f(x) = 3 + \sqrt{x+1}$???)

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Review - Solving Radical Equations - Examples

- And the algebraic solution

$$3 + \sqrt{x+1} = 2x \quad \text{where } x \geq -1$$

$$\sqrt{x+1} = 2x - 3$$

$$(\sqrt{x+1})^2 = (2x-3)^2$$

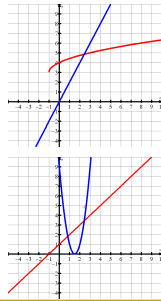
$$x+1 = 4x^2 - 12x + 9$$

$$0 = 4x^2 - 13x + 8$$

$$\therefore x = \frac{-(-13) \pm \sqrt{(-13)^2 - 4(4)(8)}}{2(4)}$$

$$\therefore x = 0.83, 2.43$$

- Explain what the term "extraneous solution" means
- Explain WHY they occur.



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Fast Five

- Define a 1:1 function
- Is $f(x) = \sqrt{x}$ a 1:1 function?
- Is $f(x) = \sqrt{x}$ the inverse FUNCTION of $g(x) = x^2$?
- Is $f(x) = x^2$ a 1:1 function?
- Explain why using the inverse function to create "an equivalent system" sometimes doesn't "work" and we get extraneous solutions
- Q? Will you get extraneous solutions when you solve cube root equations? Why? Why not?

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Radical Inequalities

- Recall that "rules" & "processes" for inequalities are sometimes different for inequalities than they are for equations
- Ex: Solve ALGEBRAICALLY

(a) $2\sqrt{x+4} \geq 6$

(b) $3\sqrt{x+2} \leq 6$

(c) $4\sqrt{4-2x} > 12$

(d) $\sqrt{x+2} < -2$

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Radical Inequalities

- Recall that "rules" & "processes" for inequalities are sometimes different for inequalities than they are for equations
- Ex: Solve GRAPHICALLY → SO WHAT'S different???

(a) $2\sqrt{x+4} \geq 6$

(b) $3\sqrt{x+2} \leq 6$

(c) $4\sqrt{4-2x} > 12$

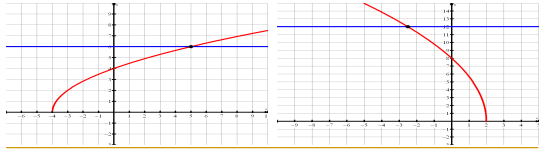
(d) $\sqrt{x+2} < -2$

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Solutions

- (a) $2\sqrt{x+4} \geq 6$
- (b) $3\sqrt{x+2} \leq 6$
- (c) $4\sqrt{4-2x} > 12$
- (d) $\sqrt{x+2} < -2$



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Radical Inequalities

- So the key point in radical inequalities is to:
 - (a) Check your solution
 - (b) check your domain

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(B) Radical Inequalities - Examples

- Solve and verify (algebraically)

$$\sqrt{x+7} > x+1$$

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(B) Radical Inequalities - Examples

- Solve and verify (algebraically)

$$\begin{aligned} \sqrt{x+7} &> x+1 \\ (\sqrt{x+7})^2 &> (x+1)^2 \\ x+7 &> x^2+2x+1 \\ 0 &> x^2+2x+1-(x+7) \\ 0 &> x^2+x-6 \\ 0 &> (x+3)(x-2) \\ \therefore x &> -3 \text{ and } x < 2 \\ \therefore -3 &< x < 2 \end{aligned}$$

Let's verify our x values:

$$\begin{aligned} x &= 2 \\ \sqrt{2+7} &= 2+1 \\ 3 &= 3 \\ x &= -3 \\ \sqrt{-3+7} &= -3+1 \\ 2 &= -2 \end{aligned}$$

So $x = -3$ doesn't verify
So $x < 2$
BUT recall domain for $x \rightarrow x > -7$
So final solution is $-7 < x < 2$

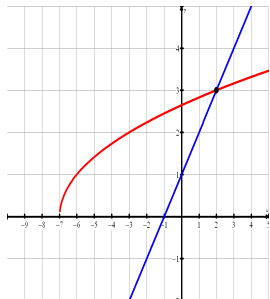
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(B) Radical Inequalities - Examples

- Let's graphically verify

$$\sqrt{x+7} > x+1$$



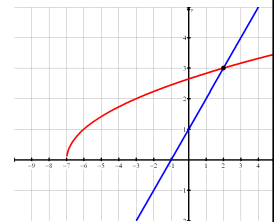
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(B) Radical Inequalities - Examples

- Looking CAREFULLY at our graphic solution
 - ➔ Is there not another logical way to eliminate the "extraneous" solution ???

$$\sqrt{x+7} > x+1$$



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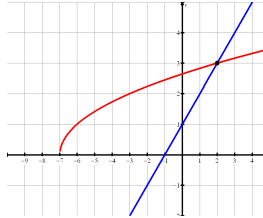
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(B) Radical Inequalities - Examples

- Looking CAREFULLY at our graphic solution → Is there not another logical way to eliminate the "extraneous" solution ???

$$\sqrt{x+7} > x+1$$

- HINT: Range of radical function is →



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(B) Radical Inequalities - Examples

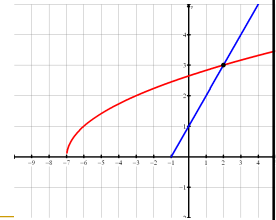
- HINT: Range of this radical function is → $y > 0$
- Which should imply that $f(x) = x+1$ should be considered for only output values of $y > 0$
- So when is $x+1 > 0$ → for $x > -1$

$$\sqrt{x+7} > x+1$$

- So our considerations for solutions should take into account $x > -1$ as well as $x > -7$

- Recall that the algebra gave us $x = -3$ and $x = 2$ → so it should be obvious that $x = -3$ is an extraneous solution

- QUESTION THOUGH → Does this domain analysis ALWAYS work??



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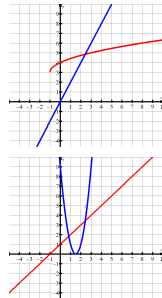
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(B) Radical Inequalities - Examples

- QUESTION THOUGH → Does this domain analysis ALWAYS work??

$$\begin{aligned} 3 + \sqrt{x+1} &> 2x \quad \text{where } x \geq -1 \\ \sqrt{x+1} &> 2x-3 \\ (\sqrt{x+1})^2 &> (2x-3)^2 \\ x+1 &> 4x^2 - 12x + 9 \\ 0 &> 4x^2 - 13x + 8 \\ \therefore x &= \frac{-(-13) \pm \sqrt{(-13)^2 - 4(4)(8)}}{2(4)} \\ \therefore x &= 0.83, 2.43 \end{aligned}$$

- NO! (as $x \geq 0$)



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(B) Radical Inequalities - Examples

- Solve & verify algebraically

(a) $\sqrt{x-5} > x+2$

(b) $\sqrt{4x+12} \leq \frac{1}{2}x+3$

(c) $\sqrt{4-2x} \geq \sqrt{x+2}+1$

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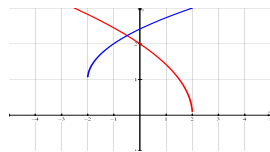
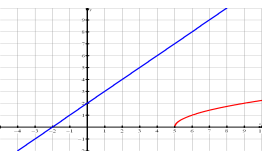
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(B) Radical Inequalities - Examples

(a) $\sqrt{x-5} > x+2$

(b) $\sqrt{4x+12} \leq \frac{1}{2}x+3$

(c) $\sqrt{4-2x} \geq \sqrt{x+2}+1$



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(B) Radical Inequalities - Examples

- Solve and verify (algebraically)

$$\sqrt{2x+4} < \sqrt{x}+1$$

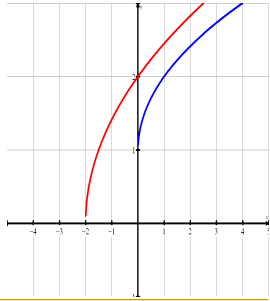
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(B) Radical Inequalities - Examples

- Solve and verify (algebraically)

$$\sqrt{2x+4} < \sqrt{x} + 1$$



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Homework

- p. 542 # 27-37 odds, 38, 45, 47, 49, 50, 52, 55, 59, 60

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