Lesson 30 – Solving Radical Equations

Math 2 Honors - Santowski

Opening Investigation

- We will investigate the idea of "equivalent systems"
- Use a graph to solve the equation $2\sqrt{x+5}=8$
- Use a graph to solve the equation $x+5=16$
- Explain what is meant by "equivalent systems" given your 2 solutions to the 2 equations

Opening Investigation

- We will investigate the idea of "equivalent systems"
- Use a graph to solve the equation $3+\sqrt{x+1}=2x$
- Use a graph to solve the equation $x+1=4x^2-12x+9$
- Is this an example of "equivalent systems" given your 2 solutions to the 2 equations?

Opening Investigation

- We will investigate the idea of "equivalent systems"
- Use a graph to solve the equation $-1+\sqrt{x}=\sqrt{2x+1}$
- Use a graph to solve the equation $4x=x^2$
- Is this an example of "equivalent systems" given your 2 solutions to the 2 equations?

(A) Solving Radical Equations

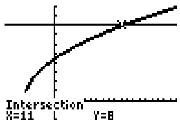
- We can prepare graphic and algebraic solutions to radical equations
- As part of our algebraic solutions, we create "equivalent" systems, which we must justify
- So our root functions have domain restrictions, we should state our restrictions at the beginning of our solutions

(A) Solving Radical Equations - Examples

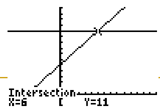
- Graphically solve $2\sqrt{x+5}=8$
- Algebraically solve $2\sqrt{x+5}=8$
- (domain for $f(x)=2\sqrt{x+5}$???)
-

(A) Solving Radical Equations - Examples

- Graphic solution is:



- Equivalent system:



- Algebra solution is:

$2\sqrt{x+5} = 8 \quad \text{where } x \geq -5$

$\sqrt{x+5} = 4$

$\left(\sqrt{x+5}\right)^2 = (4)^2$

$x+5 = 16$

$x = 11$

(A) Solving Radical Equations - Examples

- Graphically solve $3+\sqrt{x+1} = 2x$
- Algebraically solve $3+\sqrt{x+1} = 2x$
- (domain for $f(x) = 3+\sqrt{x+1}$???)
-

(A) Solving Radical Equations - Examples

- And the algebraic solution

$3+\sqrt{x+1} = 2x \quad \text{where } x \geq -1$

$\sqrt{x+1} = 2x-3$

$\left(\sqrt{x+1}\right)^2 = (2x-3)^2$

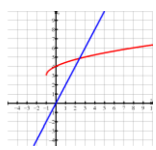
$x+1 = 4x^2 - 12x + 9$

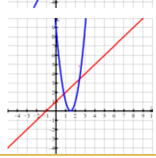
$0 = 4x^2 - 13x + 8$

$\therefore x = \frac{-(-13) \pm \sqrt{(-13)^2 - 4(4)(8)}}{2(4)}$

$\therefore x = 0.83,\ 2.43$

- Explain what the term "extraneous solution" means
- Explain WHY they occur.





(A) Solving Radical Equations - Examples

- Graphically solve $\sqrt{8x+16} = 0.5x+3$
- Algebraically solve $\sqrt{8x+16} = 0.5x+3$
- (domain for $f(x) = \sqrt{8x+16}$???)
-

(A) Solving Radical Equations - Examples

- And the algebraic solution

$\sqrt{8x+16} = 0.5x+3 \quad \text{where } x \geq -2$

$\left(\sqrt{8x+16}\right)^2 = (0.5x+3)^2$

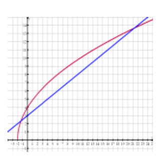
$8x+16 = 0.25x^2 + 3x + 9$

$0 = 0.25x^2 - 5x - 7$

$\therefore x = \frac{-(-5) \pm \sqrt{(-5)^2 - 4(0.25)(-7)}}{2(0.25)}$

$\therefore x = -1.314,\ 21.314$

- Explain what the term "extraneous solution" means
- Explain WHY they occur.



(A) Solving Radical Equations - Examples

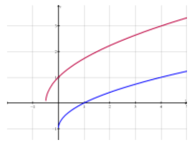
- Graphically solve $-1+\sqrt{x} = \sqrt{2x+1}$
- Algebraically solve $-1+\sqrt{x} = \sqrt{2x+1}$
- (domain for $-1+\sqrt{x}$ and $\sqrt{2x+1}$???)
-

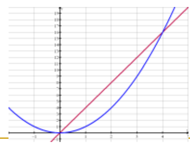
(A) Solving Radical Equations - Examples

- And the algebraic solution

$$-1+\sqrt{x}=\sqrt{2x+1} \quad \text{where } x \geq 0$$
$$\left(-1+\sqrt{x}\right)^2=\left(\sqrt{2x+1}\right)^2$$
$$1-2\sqrt{x}+x=2x+1$$
$$-2\sqrt{x}=x$$
$$\left(-2\sqrt{x}\right)^2=(x)^2$$
$$4x=x^2$$
$$0=x^2-4x$$
$$0=x(x-4)$$
$$\therefore x=0,\ 4$$

- Explain what the term “extraneous solution” means
- Explain WHY they occur.





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(B) Further Examples

- Solve and verify without a calculator:

$$\sqrt{3x+4}=\sqrt{x}+2$$
$$\sqrt{x-1}-\sqrt{x+1}=-1$$

- Are the following statements (a) always true, (b) sometimes true or (c) never true

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(B) Further Examples

- Solve algebraically and verify

$$\text{(a) } 1+\frac{\sqrt{y+4}}{\sqrt{y-3}}=\frac{7}{\sqrt{y-3}}$$
$$\text{(b) } \sqrt[3]{2x+1}=3$$
$$\text{(c) } \sqrt[3]{1-3x}-4=0$$

Math 2 Honors - Santowski 15

(B) Further Examples

- If $\sqrt{5x+11}+\sqrt{5x}=7$ then determine the value of $\sqrt{5x+11}-\sqrt{5x}$

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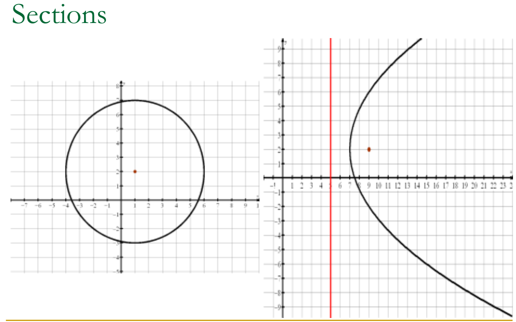
Working With Radicals – Intro to Conic Sections

- (a) What is the distance between any two given points on the Cartesian plane?
- (b) Determine the equation of the set of points that are an equidistance of 5 units from a fixed point of A(1,2)
- (c) Determine the equation of a set of points that are equidistant from the line x = 5 and the point (9,2)

Math 2 Honors - Santowski 17

Working With Radicals – Intro to Conic Sections



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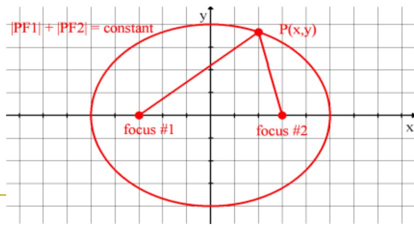
(C) Ellipses as Loci - Algebra

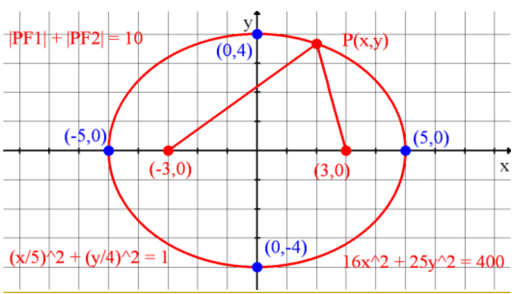
- Since we are dealing with distances, we set up our equation using the general point P(x,y), F_1 at (-3,0) and F_2 at (3,0) and the algebra follows on the next slide $|PF_1| + |PF_2| = 10$



(C) Algebraic Work

$$PF_1 + PF_2 = 10$$
$$\left(\sqrt{(x+3)^2 + y^2}\right) + \left(\sqrt{(x-3)^2 + y^2}\right) = 10$$
$$\left(\sqrt{(x+3)^2 + y^2}\right)^2 = \left(10 - \sqrt{(x-3)^2 + y^2}\right)^2$$
$$(x+3)^2 + y^2 = 100 - 20\sqrt{(x-3)^2 + y^2} + (x-3)^2 + y^2$$
$$x^2 + 6x + 9 + y^2 = 100 - 20\sqrt{(x-3)^2 + y^2} + x^2 - 6x + 9 + y^2$$
$$20\sqrt{(x-3)^2 + y^2} = 100 - 12x$$
$$\left(5\sqrt{(x-3)^2 + y^2}\right)^2 = (25 - 3x)^2$$
$$25\left(x^2 - 6x + 9 + y^2\right) = 625 - 150x + 9x^2$$
$$25x^2 - 9x^2 - 150x + 150x + 25y^2 = 625 - 225$$
$$16x^2 + 25y^2 = 400$$
$$\left(\frac{16x^2}{400}\right) + \left(\frac{25y^2}{400}\right) = \left(\frac{x^2}{25}\right) + \left(\frac{y^2}{16}\right) = \left(\frac{x}{5}\right)^2 + \left(\frac{y}{4}\right)^2 = 1$$

(D) Graph of the Ellipse



Homework

- p. 542 # 13-23 odds, 24, 39, 41, 43, 53, 57, 61-63