

Lesson 26 – Solving Rational Inequalities

Math 2 Honors

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Fast Five

▶ Solve $\frac{x}{2x-1} \leq 1$

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Lesson Objectives

- ▶ 1. Review multiple methods for solving inequalities
- ▶ 2. Review the “sign chart” as an algebraic “tool” to help solve rational inequalities
- ▶ 3. Solve rational inequalities

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Fast Five

- ▶ You will work with the equation $r(x) = \frac{x^2+1}{x^2-x-2}$
- ▶ Perform a long division (will synthetic not work?!) and interpret the quotient
- ▶ Evaluate and interpret $\lim_{x \rightarrow \infty} \frac{x^2+1}{x^2-x-2}$
- ▶ Solve and interpret $\frac{x^2+1}{x^2-x-2} = 1$

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Fast Five - Answers

- ▶ You will work with the equation $r(x) = \frac{x^2+1}{x^2-x-2}$
- ▶ Perform a long division (will synthetic not work?) and interpret the quotient $r(x) = \frac{x^2+1}{x^2-x-2} = 1 + \frac{x+3}{x^2-x-2}$
- ▶ Evaluate and interpret $\lim_{x \rightarrow \infty} \frac{x^2+1}{x^2-x-2} = \lim_{x \rightarrow \infty} \left(1 + \frac{x+3}{x^2-x-2} \right) = 1$
- ▶ Solve and interpret which is where the rational function crosses its horizontal asymptote!! $1 + \frac{x+3}{x^2-x-2} = 1$
 $\therefore \frac{x+3}{x^2-x-2} = 0$
 $\therefore x = -3$

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(A) Solving Rational Inequalities -

- ▶ We will work with the rational inequality $\frac{x+2}{x-4} < 2x-3$
- ▶ (1) Interpret the MEANING of the inequality
- ▶ (2) Set up a table of values and answer the question from the data table
- ▶ (3) Prepare a graph BY HAND and answer the question
- ▶ (4) Graph on the calculator
- ▶ (5) Solve using algebra (GASP!!!!)

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(A) Solving Rational Inequalities - ANS

▶ We will work with the rational inequality $\frac{x+2}{x-4} < 2x-3$

▶ (1) Interpret the MEANING of the inequality → when is the line $g(x) = 2x-3$ higher than the rational function

$$f(x) = \frac{x+2}{x-4}$$

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(A) Solving Rational Inequalities - ANS

▶ We will work with the rational inequality

$$\frac{x+2}{x-4} < 2x-3$$

▶ (2) Set up a table of values and answer the question from the data table

x	y ₁	y ₂
-2	0	-7
-1	-0.2	-5
0	-0.5	-3
1	-1.0	-1
2	-2.0	1
3	-5.0	3
4	undef	5
5	7.0	7
6	4.0	9
7	3.0	11
8	2.5	13
9	2.2	15
10	2.0	17

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(A) Solving Rational Inequalities - ANS

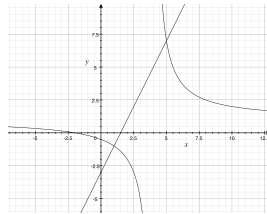
▶ We will work with the rational inequality

$$\frac{x+2}{x-4} < 2x-3$$

▶ (3) Prepare a graph BY HAND and answer the question

▶ Recall long division (or synthetic in this case)

$$\frac{x+2}{x-4} = 1 + \frac{6}{x-4}$$

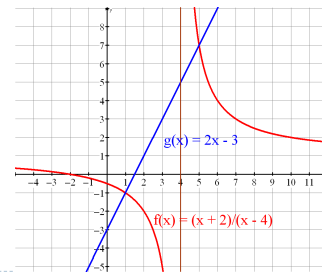


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(A) Solving Rational Inequalities - ANS

▶ We will graph using technology $\frac{x+2}{x-4} < 2x-3$



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(A) Solving Rational Inequalities - ANS

▶ We will solve using algebra ▶ Now that we have done all the algebra and factored everything, use a sign chart to solve the inequality:

$$\begin{aligned} \frac{x+2}{x-4} < 2x-3; x \neq 4 \\ \frac{x+2}{x-4} < \frac{2x-3}{1} \\ \frac{x+2}{x-4} \left(\frac{1}{1} \right) < \frac{(x-4)(2x-3)}{(x-4)(1)} \\ 0 < \frac{(x-4)(2x-3)}{(x-4)(1)} - \frac{(x+2)(1)}{(x-4)(1)} \\ 0 < \frac{(2x^2-11x+12)}{(x-4)} - \frac{(x+2)}{(x-4)} \\ 0 < \frac{(2x^2-12x+10)}{(x-4)} \\ 0 < \frac{2(x-5)(x-1)}{(x-4)} \end{aligned}$$

	$(-\infty, 1)$	$(1, 4)$	$(4, 5)$	$(5, \infty)$
$(x-5)$	-ve	-ve	-ve	+ve
$(x-1)$	-ve	+ve	+ve	+ve
$(x-4)$	-ve	-ve	+ve	+ve
R(x)	-ve	+ve	-ve	+ve

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(A) Solving Rational Inequalities - ANS

▶ We will work with the rational inequality $\frac{x+2}{x-4} < 2x-3$

▶ So regardless of how we set it up, we come up with the same solution → $\{x \in \mathbb{R} \mid 1 < x < 4 \text{ or } x > 5\}$

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(B) Further Examples

► To illustrate and emphasize the algebraic methods (and as a way of incorporating past algebra skills), solve the following rational inequalities:

(i) $\frac{x}{2x-1} \leq 1$

(ii) $\frac{x-2}{2(x-3)} > \frac{x}{x+3}$

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(B) Further Examples - Solutions

► Here is the algebra

► Here is the sign chart:

(i) $\frac{x}{2x-1} \leq 1$
 $\left(\frac{x}{2x-1}\right)\left(\frac{1}{1}\right) \leq \left(\frac{1}{1}\right)\left(\frac{2x-1}{2x-1}\right); x \neq \frac{1}{2}$
 $\frac{x}{2x-1} \leq \frac{2x-1}{2x-1}$
 $0 \leq \frac{2x-1}{2x-1} - \frac{x}{2x-1}$
 $0 \leq \frac{x-1}{2x-1}; x \neq \frac{1}{2}$

	$X < \frac{1}{2}$	$(\frac{1}{2}, 1)$	$X > 1$
$x - 1$	-ve	-ve	+ve
$2x - 1$	-ve	+ve	+ve
$R(x)$	+ve	-ve	+ve

► So the solution is that $x > 1$ or $x < \frac{1}{2}$

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(B) Further Examples - Solutions

► Here is the algebra

► Here is the sign chart:

$\frac{x-2}{2(x-3)} > \frac{x}{x+3}$
 $\frac{(x-2)(x+3)}{2(x-3)(x+3)} > \frac{2x(x-3)}{2(x-3)(x+3)}$
 $0 > \frac{2x^2 - 6x}{2(x-3)(x+3)} - \frac{x^2 + x - 6}{2(x-3)(x+3)}$
 $0 > \frac{x^2 - 7x + 6}{2(x-3)(x+3)}$
 $0 > \frac{(x-6)(x-1)}{2(x-3)(x+3)}$

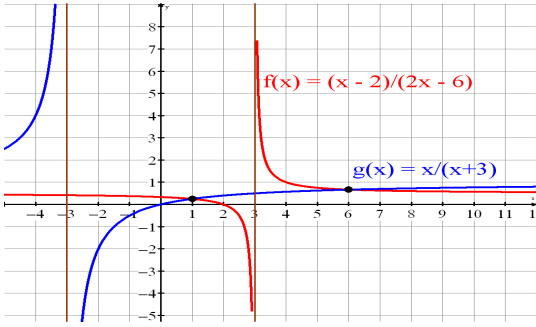
	$X < -3$	$(-3, 1)$	$(1, 3)$	$(3, 6)$	$X > 6$
$x-6$	-ve	-ve	-ve	-ve	+ve
$x-1$	-ve	-ve	+ve	+ve	+ve
$x-3$	-ve	-ve	-ve	+ve	+ve
$x+3$	-ve	+ve	+ve	+ve	+ve
$R(x)$	+ve	-ve	+ve	-ve	+ve

► So the solution is that $x > 1$ or $x < \frac{1}{2}$

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(B) Further Examples - Solutions



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Homework

► p. 518 # 29-41 odd, 43-46, 51-53, 55

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