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# Lesson 25 - Solving Rational Equations

Math 2Honors - Santowski

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## FAST FIVE

- Predict the end behaviour of  $f(x) = \frac{x}{x-6}$
- Predict the VA and NVA of  $f(x) = \frac{x}{x-6}$
- Predict the end behaviour of  $g(x) = \frac{1}{x-4}$
- Predict the VA and NVA of  $g(x) = \frac{1}{x-4}$
- HOPEFULLY, as the lesson progresses, you will see the connection between SOLVING EQUATIONS and KNOWING WHAT GRAPHS LOOK LIKE

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## Lesson Objectives

- Review strategies for solving equations and apply them to rational equations (algebraic, graphic, numeric & checking solutions)
- HOPEFULLY, as the lesson progresses, you will see the connection between SOLVING EQUATIONS and KNOWING WHAT GRAPHS LOOK LIKE
- Solve equations in the context of applications & modeling

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## (A) Review

- We can solve ANY given equation in a variety of ways:
  - (a) Numeric
  - (b) Graphic
  - (c) Algebraic

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## (B) Solving Numerically - Example 1

- Solve the following equation, given the following table of values:
 
$$\frac{x}{x-6} = \frac{1}{x-4}$$
- The table of values shows that the solution is .....??
- Since the table of values only lies between  $\{x \in \mathbb{I} \mid -4 < x < 5\}$ , are there OTHER solutions:
  - (i) beyond either  $x = -4$  or  $x = 5$ ?
  - (ii) are there non-integral solutions??

x	f(x)	g(x)
-4	0.4	-0.125
-3	0.333...	-0.1429...
-2	0.25	-0.16666..
-1	0.1429...	-0.2
0	0	-0.25
1	-0.2	-0.3333..
2	-0.5	-0.5
3	-1	-1
4	-2	Undefined
5	-5	1

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## (B) Solving Numerically - Example 1

- Solve the following equation, given the following table of values:
 
$$\frac{x}{x-6} = \frac{1}{x-4}$$
- The table of values shows that the solution is .....??
- Since the table of values only lies between  $\{x \in \mathbb{I} \mid -4 < x < 5\}$ , are there OTHER solutions:
  - (i) beyond either  $x = -4$  or  $x = 5$ ?
  - (ii) are there non-integral solutions??

x	f(x)	g(x)
5	-5	1
6	Error	0.5
7	7	0.3333...
8	4	0.25
9	3	0.2
10	2.5	0.1666..
11	2.2	0.1428...
12	2	0.125
13	1.875...	0.1111...
14	1.75	0.1

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### (B) Solving Numerically - Example 1 Solution

- Solve  $\frac{x}{x-6} = \frac{1}{x-4}$
- The table of values shows that the solution is  $x = 2$  and  $x = 3$  since the 2 functions have the same function values at these x values
- (i) beyond  $x = -4$  & 5 (No since end behaviours ( $\pm\infty$  are different 1 and 0)
- (ii) are there non-integral solutions?? (No, since one function is always above the other in each integral interval)

x	f(x)	g(x)
-4	0.4	-0.125
-3	0.333...	-0.1429...
-2	0.25	-0.16666..
-1	0.1429...	-0.2
0	0	-0.25
1	-0.2	-0.3333..
2	-0.5	-0.5
3	-1	-1
4	-2	Undefined
5	-5	1

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### (B) Solving Numerically - Example 2

- Solve the following equation, given the following table of values:

$$\frac{x}{x-6} = \frac{1}{x-4.25}$$

- The table of values shows that the solution is .....??
- Since the table of values only lies between  $\{x \in \mathbb{R} \mid -4 < x < 5\}$ , are there OTHER solutions:
- (i) beyond either  $x = -4$  or  $x = 5$ ?
- (ii) are there non-integral solutions??

x	f(x)	g(x)
-4	0.4	-0.1212...
-3	0.333...	-0.1379...
-2	0.25	-0.16
-1	0.1429...	-0.1905...
0	0	-0.2353...
1	-0.2	-0.3077...
2	-0.5	-0.4444...
3	-1	-0.8
4	-2	-4
5	-5	1.3333...

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### (C) Solving Graphically

- Solve by graphing:  $\frac{x}{x-6} = \frac{1}{x-4}$
- When we solve by graphing, WHAT do we look for???

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### (C) Solving Graphically

- Solve by graphing:
- When we solve by graphing, WHAT do we look for???

$$\frac{x}{x-6} = \frac{1}{x-4}$$

- We are looking for the x coordinate of the intersection point

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### (C) Solving Graphically

- Solve by graphing:  $\begin{cases} f(x) = \frac{x}{x-6} \\ g(x) = \frac{1}{x-4} \end{cases}$
- When we solve by graphing, WHAT do we look for???

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### (C) Solving Graphically

- Solve by graphing:
- When we solve by graphing, WHAT do we look for???

$$\begin{cases} f(x) = \frac{x}{x-6} \\ g(x) = \frac{1}{x-4} \end{cases}$$

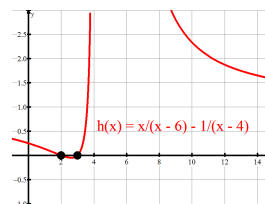
- We are looking for the x and y co-ordinates of the intersection point **(2, -0.5)** and **(3, -1)**

### (C) Solving Graphically

- Solve by graphing:  $\frac{x}{x-6} = \frac{1}{x-4}$
  - But we will set up an **ALTERNATIVE** graphing approach  $\rightarrow$  what's different NOW???
- $$\frac{x}{x-6} - \frac{1}{x-4} = 0$$
- When we solve by graphing with this alternative method, **WHAT** do we look for???

### (C) Solving Graphically

- Solve by graphing:
  - When we solve by graphing, **WHAT** do we look for???
- $$\frac{x}{x-6} - \frac{1}{x-4} = 0$$
- We are looking for the **x** intercepts or roots or zeroes of the rational function
- $$\frac{x}{x-6} - \frac{1}{x-4} = h(x)$$



### (D) Solving Algebraically

- Solve for  $x$  algebraically  $\frac{x}{x-6} = \frac{1}{x-4}$
- 
- So now we must firstly consider **RESTRICTIONS** upon the independent variable,  $x \rightarrow$  **WHY**???

### (D) Solving Algebraically

- Solve for  $x$  algebraically  $\frac{x}{x-6} = \frac{1}{x-4}$
  -
- $$\frac{x}{x-6} = \frac{1}{x-4}$$
- $$\left(\frac{x-4}{x-4}\right)\left(\frac{x}{x-6}\right) = \left(\frac{1}{x-4}\right)\left(\frac{x-6}{x-6}\right); x \neq 4, 6$$
- $$x(x-4) = 1(x-6)$$
- $$x^2 - 4x - x + 6 = 0$$
- $$x^2 - 5x + 6 = 0$$
- $$(x-3)(x-2) = 0$$
- $$\therefore x = 2, 3 \text{ but } x \neq 4, 6$$

### (D) Solving Algebraically

- in our algebraic solution, we algebraically created an **EQUIVALENT** system
- $$\frac{x}{x-6} = \frac{1}{x-4}$$
- became
- $$x^2 - 5x + 6 = 0$$
- EXPLAIN** what we mean by an **EQUIVALENT** system????

### (D) Solving Algebraically

- in our algebraic solution, we algebraically created an **EQUIVALENT** system
- $$\frac{x}{x-6} = \frac{1}{x-4}$$
- became
- $$x^2 - 5x + 6 = 0$$
- EXPLAIN** what we mean by **EQUIVALENT** systems????
  - Now explain **WHY** we need to check our algebraic solutions?



### (E) Practice

- Now that we have reviewed 3 different solving strategies, work through the following examples using the given strategies:

- (i) algebraic and graphic  $\frac{x}{x-3} + \frac{2x}{x+3} = \frac{18}{x^2-9}$
- (ii) algebraic and numeric  $\frac{x}{x-2} + \frac{x}{x-3} = \frac{3}{x^2-5x+6}$

### (F) Applications of Rational Equations

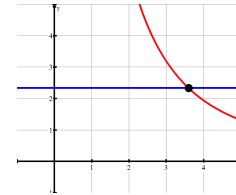
- Ex 1. Rowing at 8 km/h, in still water, Rima and Bhanu take 16 h to row 39 km down a river and 39 km back. Find the speed of the current to two decimal places.
- Ex 2. Jaime bought a case of concert T-shirts for \$450. She kept two for herself and sold the rest for \$560, making a profit of \$10 on each shirt. How many shirts were in the case?

### (F) Applications of Rational Equations

- Mr S. and Mr. E set off at the same time on a 30K run. Mr. E. runs 1.4 km/h faster than Mr. S. Mr. E. finishes 2 hrs ahead of Mr. S. in spite of a 20 minute rest in the race. How fast was each one running and how long did it take each person to run the race?

### (F) Applications of Rational Equations

- Mr S. and Mr. E set off at the same time on a 30K walk. Mr. E. runs 1.4 km/h faster than Mr. S. Mr. E. finishes 2 hrs ahead of Mr. S. in spite of a 20 minute rest in the race. How fast was each one running and how long did it take each person to run the race?



- Equation is  $\frac{30}{v_s} - \frac{30}{v_s + 1.4} = 2\frac{1}{3}$

- Solution is  $v_s = 3.6$  km/h and it took me 8 hours & 20 min

### (G) Homework

- Textbook, p. 517 # 9-27 odd, 47-50, 54, 56-57