Lesson 20 – Solving Polynomial Equations

Math 2 Honors - Santowski

Lesson Objectives

- Mastery of the factoring of polynomials using the algebraic processes of long & synthetic division
- Mastery of the algebraic processes of solving polynomial equations by factoring (Factor Theorem)
- Investigate how equations can be factored and solved graphically, numerically, and by technology
- Reinforce the understanding of the connection between factors and roots

(A) Roots & Factors

- In our work with quadratics, we saw the "factored" form or "intercept" form of a quadratic equation/expression
- i.e. $f(x) = x^2 – x – 2 = (x – 2)(x + 1)$ ➔ factored form of eqn
- So when we solve $f(x) = 0$ ➔ $0 = (x – 2)(x + 1)$, we saw that the zeroes/x-intercepts/roots were $x = 2$ and $x = -1$
- So we established the following connection:
- Factors ➔ $(x - 2)$ and $(x – 1)$
- Roots ➔ $x = 2$ and $x = 1$
- So we will now reiterate the following connections:
- If $(x – R)$ is a ***factor*** of P(x), then $x = R$ is ***root*** of P(x)
  AND THE CONVERSE
- If $x = R$ is a ***root*** of P(x), then $(x – R)$ is a ***factor*** of P(x)

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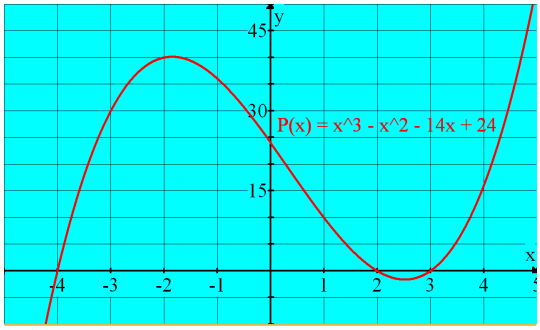
(B) Review

- Divide $x^3 – x^2 - 14x + 24$ by $x - 2$ and notice the remainder
- Then evaluate P(2). What must be true about $(x - 2)$?
- Divide $x^3 – x^2 - 14x + 24$ by $x + 3$ and notice the remainder
- Then evaluate P(-3). What must be true about $(x+3)$?
- Now graph $f(x) = x^3 – x^2 - 14x + 24$ and see what happens at $x = 2$ and $x = -3$
- So our conclusion is that $x - 2$ is a factor of $x^3 – x^2 - 14x + 24$, whereas $x + 3$ is not a factor of $x^3 – x^2 - 14x + 24$

(B) Review – Graph of $P(x) = x^3 – x^2 - 14x + 24$



(C) The Factor Theorem

- We can use the ideas developed in the review to help us to draw a connection between the polynomial, its factors, and its roots.
- What we have seen in our review are the key ideas of the Factor Theorem - in that if we know a root of an equation, we know a factor and the converse, that if we know a factor, we know a root.
- The Factor Theorem is stated as follows: ***x - a* is a factor of *f(x)* if and only if *f(a) = 0***. To expand upon this idea, we can add that ***ax - b* is a factor of *f(x)* if and only if *f(b/a) = 0*.**
- Working with polynomials, $(x + 1)$ is a factor of $x^2 + 2x + 1$ because when you divide $x^2 + 2x + 1$ by $x + 1$ you get a 0 remainder and when you substitute $x = -1$ into $x^2 + 2x + 1$, you get 0

(D) Examples

- ex 1. Show that x - 2 is a factor of $x^3 - 7x + 6$
- ex. 2. Show that -2 is a root of $2x^3 + x^2 - 2x + 8 = 0$. Find the other roots of the equation. (Show with GC)
- ex. 3. Factor $x^3 + 1$ completely
- Ex. 4. Factor $x^3 - 1$ completely
- ex. 4. Is $x - \sqrt{2}$ a factor of $x^4 - 5x^2 + 6$?

(D) Further Examples

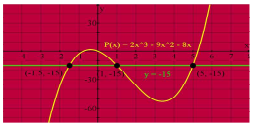
- Ex 1 ➔ Factor $P(x) = 2x^3 - 9x^2 + 7x + 6$ & then solve $P(x) = 0$
- Ex 2 ➔ Solve $3x^3 - 7x^2 + 8x - 2 = 0$
- ex 3 ➔ Graph $f(x) = 3x^3 + x^2 - 22x - 24$ using intercepts, points, and end behaviour. Approximate turning points, max/min points, and intervals of increase and decrease.

(D) Further Examples

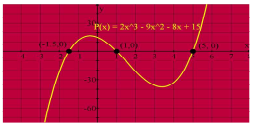
- ex.1 Solve $2x^3 - 9x^2 - 8x = -15$ and then show on a GDC
- ie. Solve the system $\begin{cases} y = 2x^3 - 9x^2 - 8x \\ y = -15 \end{cases}$
- ex 2. Solve $2x^3 + 14x - 20 = 9x^2 - 5$ and then show on a GDC
- ie. Solve the system $\begin{cases} y = 2x^3 + 14x - 20 \\ y = 9x^2 - 5 \end{cases}$
- ex. 3 Solve $x^4 - x^3 - 7x^2 + 13x - 6 = 0$ then graph using roots, points, end behaviour. Approximate turning points, max/min points, and intervals of increase and decrease (HINT for domain of solution ➔ use RRT}

(D) Further Examples - Solutions

- Solve $2x^3 - 9x^2 - 8x = -15$ and then show on a GDC
- Now graph both
- g(x) = $2x^3 - 9x^2 - 8x$ and then
- h(x) = -15 and find intersection

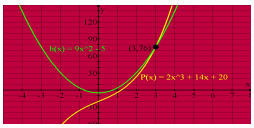


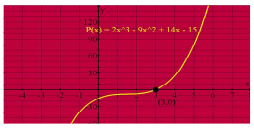
- Then graph:
- f(x) = $2x^3 - 9x^2 - 8x + 15$



(D) Further Examples - Solutions

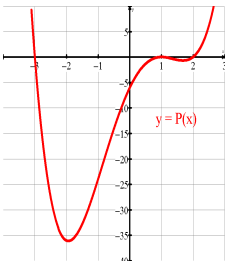
- Solve $2x^3 + 14x - 20 = 9x^2 - 5$ and then show on a GDC
- Explain that different solution sets are possible depending on the number set being used (real or complex)





(D) Further Examples - Solutions

- Solve $x^4 - x^3 - 7x^2 + 13x - 6 = 0$
- Then graph using roots, points, end behaviour. Approximate turning points, max/min points, and intervals of increase and decrease.
- $P(x) = (x - 1)^2(x + 3)(x - 2)$

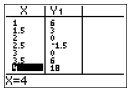


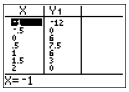
(E) Solving & Factoring on the TI-84

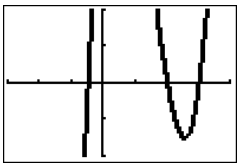
- Factor & Solve the following:
- $0 = 2x^3 - 9x^2 + 7x + 6$
- $3x^3 - 7x^2 + 8x - 2 = 0$
- $x^4 - x^3 - 7x^2 + 13x - 6 = 0$
- $2x^3 - 9x^2 - 8x = -15$
- $2x^3 + 14x - 20 = 9x^2 - 5$

(E) Solving & Factoring on the TI-84

- Factor & Solve the following:
- $0 = 2x^3 - 9x^2 + 7x + 6$ ➔ roots at x = -0.5,2,3 ➔ would imply factors of (x – 2), (x – 3) and (x + ½) ➔ P(x) = 2(x + ½)(x – 2)(x – 3)
- So when factored P(x) = (2x + 1)(x – 2)(x – 3)

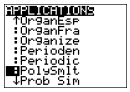


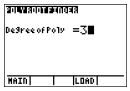


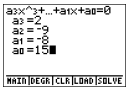


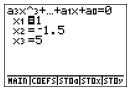
(E) Solving & Factoring on the TI-84

- Solve $2x^3 - 9x^2 - 8x = -15$ turn it into a "root" question ➔ i.e Solve P(x) = 0 ➔ Solve $0 = 2x^3 - 9x^2 - 8x + 15$









(F) Examples - Applications

- ex 5. You have a sheet of paper 30 cm long by 20 cm wide. You cut out the 4 corners as squares and then fold the remaining four sides to make an open top box.
  - (a) Find the equation that represents the formula for the volume of the box.
  - (b) Find the volume if the squares cut out were each 2 cm by 2 cm.
  - (c) What are the dimensions of the squares that need to be removed if the volume is to be 1008 cm^3?

(F) Examples - Applications

- The volume of a rectangular-based prism is given by the formula $V(x) = -8x + x^3 - 5x^2 + 12$
  - (i) Express the height, width, depth of the prism in terms of *x*
  - (ii) State any restrictions for *x*. Justify your choice
  - (iii) what would be the dimensions on a box having a volume of 650 cubic units?
  - (iv) now use graphing technology to generate a reasonable graph for V(x). Justify your window/view settings

(F) Examples - Applications

- The equation $p(m) = 6m^5 - 15m^4 - 10m^3 + 30m^2 + 10$ relates the production level, p, in thousands of units as a function of the number of months of labour since October, m.
- Use graphing technology to graph the function and determine the following:
  - maximums and minimums. Interpret in context
  - Intervals of increase and decrease. Interpret
  - Explain why it might be realistic to restrict the domain. Explain and justify a domain restriction
  - Would $0 \le m \le 3$ be a realistic domain restriction?
- Find when the production level is 15,500 units (try this one algebraically as well)

(G) Internet Links - Factor Theorem

- The Factor Theorem from the Math Page
- College Algebra Tutorial on The Factor Theorem
- The Factor Theorem from Purple Math

(G) Internet Links - Solving Polynomials

- Finding Zeroes of Polynomials from WTAMU
- Finding Zeroes of Polynomials Tutorial #2 from WTAMU
- Solving Polynomials from Purple Math

(H) Homework

- Textbook S7.4
- P453, Q15,21,27,29,33,35,39,43,49,51,56,57,58,