Lesson: L18

Topic: Investigating Polynomial Functions

(A) Terminology

Polynomials: an expression in the form of $a_n x^n + a_{n-1} x^{n-1} + a_{n-2} x^{n-2} + \ldots\ldots + a_2 x^2 + a_1 x + a_0$

where a_0, a_1, ...a_n are real numbers and n is a natural number

leading coefficient: the coefficient of the term with the highest power

degree: the value of the highest exponent on the variable

Polynomial Functions: a function whose equation is defined by a polynomial in one variable:

ex: $f(x) = a_n x^n + a_{n-1} x^{n-1} + a_{n-2} x^{n-2} + \ldots\ldots + a_2 x^2 + a_1 x + a_0$

Standard Form: the function is expressed such that the terms are written in descending order of the exponents

Domain: the set of all possible *x* values (independent variable) in a function

Range: the set of all possible function values (dependent variable, or *y* values)

to evaluate a function: substituting in a value for the variable and then determining a function value. Ex f(3)

finite differences: subtracting consecutive *y* values or subsequent *y* differences

zeroes, roots, x-intercepts: where the function crosses the *x* axes

y-intercepts: where the function crosses the *y* axes

direction of opening: in a quadratic, curve opens up or down

symmetry: whether the graph of the function has "halves" which are mirror images of each other

turning point: points where the direction of the function changes

maximum: the highest point on a function

minimum: the lowest point on a function

local vs absolute: a max can be a highest point in the entire domain (absolute) or only over a specified region within the domain (local). Likewise for a minimum.

increase: the part of the domain (the interval) where the function values are getting larger as the independent variable gets higher; if $f(x_1) < f(x_2)$ when $x_1 < x_2$; the graph of the function is going up to the right (or down to the left)

decrease: the part of the domain (the interval) where the function values are getting smaller as the independent variable gets higher; if $f(x_1) > f(x_2)$ when $x_1 < x_2$; the graph of the function is going up to the left (or down to the right)

"end behaviour": describing the function values (or appearance of the graph) as *x* values getting infinitely large or infinitely large on the negative side ($x \to +\infty$ or $x \to -\infty$)

(B) Types of Polynomial Functions

(i) Linear: Functions that generate graphs of straight lines, polynomials of degree one => $f(x) = a_1 x^1 + a_0$ or more commonly written as $y = mx + b$, or $Ax + By + C = 0$, or $y = k(x - s)$

(ii) Quadratic: Functions that generate graphs of parabolas; polynomials of degree two => $f(x) = a_2 x^2 + a_1 x^1 + a_0$ or $y = Ax^2 + Bx + C$ or $y = a(x-s)(x-t)$ or $y = a(x - h)^2 + k$

(iii) Cubic polynomials of degree 3

(iv) Quartic: polynomials of degree 4

(v) polynomials of degree 5

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(C) Investigating Characteristics of Cubic Functions

	$f(x) = x^3 - 5x^2 + 3x + 4$	$f(x) = -2x^3 + 8x^2 - 5x + 3$	$f(x) = -3x^3 - 15x^2 - 9x + 27$
leading coefficient			
degree			
domain			
range			
evaluating f(-2)			
zeroes or roots			
possible number of roots			
y-intercept			
symmetry			
turning points			
maximum values (local and absolute)			
minimum values (local and absolute)			
intervals of increase			
intervals of decrease			
end behaviour (+x)			
end behaviour (-x)			

Conclusions for Cubic Functions:

1. Describe the general shape of a cubic function
2. Describe how the graph of a cubic function with a positive leading coefficient is different than a cubic with a positive leading coefficient
3. What does a_0 represent on the graph of a cubic?
4. How many real roots do/can cubic functions have?
5. How many complex roots do/can cubic functions have?
6. How many turning points do/can cubic functions have?
7. How many intervals of increase do/can cubic functions have?
8. How many intervals of decrease do/can cubic functions have?
9. Describe the end behaviour ($\pm x$) of a cubic with a (i) positive (ii) negative leading coefficient
10. Are cubic functions symmetrical? (You may need to investigate further)

(ii) Quartic Functions

	$f(x)= -2x^4-4x^3+3x^2+6x+9$	$f(x)= x^4-3x^3+3x^2+8x+5$	$f(x) = \frac{1}{2}x^4-2x^3+x^2+x+1$
leading coefficient			
degree			
domain			
range			
evaluating f(-2)			
zeroes or roots			
number of roots			
y-intercept			
symmetry			
turning points			
maximum values (local and absolute)			
minimum values (local and absolute)			
intervals of increase			
intervals of decrease			
end behaviour (+x)			
end behaviour (-x)			

Conclusions for Quartic Functions

1. Describe the general shape of a quartic function
2. Describe how the graph of a quartic function with a positive leading coefficient is different than a quartic with a positive leading coefficient
3. What does a_0 represent on the graph of a quartic?
4. How many real roots do/can quartic functions have?
5. How many complex roots do/can quartic functions have?
6. How many turning points do/can quartic functions have?
7. How many intervals of increase do/can quartic functions have?
8. How many intervals of decrease do/can quartic functions have?
9. Describe the end behaviour ($\pm x$) of a quartic with a (i) positive (ii) negative leading coefficient
10. Are quartic functions symmetrical? (You may need to investigate further)

(v) Quintic Functions

	$f(x)= x^5+7x^4-3x^3-18x^2-20$	$f(x)= -\frac{1}{4}x^5+ 2x^4-3x^3+3x^2+8x+5$	$f(x)=(x^2-1)(x^2-4)(x+3)$
leading coefficient			
degree			
domain			
range			
evaluating f(-2)			
zeroes or roots			
number of roots			
y-intercept			
symmetry			
turning points			
maximum values (local and absolute)			
minimum values (local and absolute)			
intervals of increase			
intervals of decrease			
end behaviour (+x)			
end behaviour (-x)			

Conclusions for Quintic Functions

1. Describe the general shape of a quintic function
2. Describe how the graph of a quintic function with a positive leading coefficient is different than a quintic with a positive leading coefficient
3. What does a_0 represent on the graph of a quintic?
4. How many real roots do/can quintic functions have?
5. How many complex roots do/can quintic functions have?
6. How many turning points do/can quintic functions have?
7. How many intervals of increase do/can quintic functions have?
8. How many intervals of decrease do/can quintic functions have?
9. Describe the end behaviour ($\pm x$) of a quintic with a (i) positive (ii) negative leading coefficient
10. Are quintic functions symmetrical? (You may need to investigate further)

(D) Examples of Further Work with Polynomials

1. Expand and simplify $h(x) = (x - 1)(x + 3)^2(x + 2)$.

2. Where are the zeroes of h(x)?

3. Predict the end behaviour of h(x).

4. Predict the shape/appearance of h(x).

5. Use a table of values to find additional points on h(x) and sketch a graph.

6. Predict the intervals of increase and decrease for h(x).

7. Estimate where the turning points of h(x) are. Are the max/min? and local/absolute if domain was [-4,1]

8. Sketch a graph of the polynomial function which has a degree of 4, a negative leading coefficient, 3 zeroes and 3 turning points

9. Equation writing: Determine the equation of a cubic whose roots are -2, 3,4 and f(5) = 28

10. Prepare a table of differences for $f(x) = -2x^3 + 4x^2 - 3x - 2$. What is the constant difference and when does it occur? Is there a relationship between the equation and the constant difference? Can you predict the constant difference for $g(x) = 4x^4 + x^3 - x^2 + 4x - 5$?