Lesson 18 – Investigating Polynomial Functions

Math 2 Honors – Santowski

Lesson Objectives

- Describe the key features of the graph of a polynomial function given in root form & standard form
- Graph a cubic, quartic and quintic function on a graphing calculator
- Use the GDC to analyze the features of the polynomial function

(A) Terminology

- **Polynomials**: an expression in the form of $a_nx^n + a_{n-1}x^{n-1} + a_{n-2}x^{n-2} + \ldots\ldots + a_2x^2 + a_1x + a_0$ where $a_0, a_1, \ldots a_n$ are real numbers and n is a natural number
- **Polynomial Functions**: a function whose equation is defined by a polynomial in one variable:
- ex: $f(x) = a_nx^n + a_{n-1}x^{n-1} + a_{n-2}x^{n-2} + \ldots\ldots + a_2x^2 + a_1x + a_0$
- **leading coefficient**: the coefficient of the term with the highest power
- **degree**: the value of the highest exponent on the variable
- **Standard Form**: the function is expressed such that the terms are written in descending order of the exponents

(A) Terminology

- **Domain**: the set of all possible *x* values (independent variable) in a function
- **Range**: the set of all possible function values (dependent variable, or *y* values)
- **to evaluate a function**: substituting in a value for the variable and then determining a function value. Ex f(3)
- **zeroes, roots, x-intercepts**: where the function crosses the *x* axes
- **y-intercepts**: where the function crosses the *y* axes
- **direction of opening**: parts of the curve opens up or down
- **symmetry**: whether the graph of the function has "halves" which are mirror images of each other

(A) Terminology

- **turning point**: points where the direction of the function changes
- **maximum**: the highest point on a function
- **minimum**: the lowest point on a function
- **local vs absolute**: a max can be a highest point in the entire domain (absolute) or only over a specified region within the domain (local). Likewise for a minimum.
- **increase**: the part of the domain (the interval) where the function values are getting larger as the independent variable gets higher; if $f(x_1) < f(x_2)$ when $x_1 < x_2$; the graph of the function is going up to the right (or down to the left)
- **decrease**: the part of the domain (the interval) where the function values are getting smaller as the independent variable gets higher; if $f(x_1) > f(x_2)$ when $x_1 < x_2$; the graph of the function is going up to the left (or down to the right)
- **"end behaviour"**: describing the function values (or appearance of the graph) as *x* values getting infinitely large positively or infinitely large negatively

(B) Types of Polynomial Functions

- (i) **Linear Functions** that generate graphs of straight lines, polynomials of degree one => $f(x) = a_1x_1 + a_0$ or more commonly written as $y = mx + b$, or $Ax + By + C = 0$, or $y = k(x - s)$
- (ii) **Quadratic: Functions** that generate graphs of parabolas; polynomials of degree two ➔ $f(x) = a_2x^2 + a_1x^1 + a_0$ or $y = Ax^2 + Bx + C$ or $y = a(x-s)(x-t)$ or $y = a(x - h)^2 + k$
- (iii) **Cubic** polynomials of degree 3
- (iv) **Quartic**: polynomials of degree 4
- (v) **Quintic**: polynomials of degree 5

(C) Investigating Characteristics of Polynomial Functions

- We can complete the following analysis for polynomials of degrees 3 through 5 and then make some generalizations or summaries:
- In order to carry out this investigation, usea GDC
- You may also use the following program from AnalyzeMath

(i) Cubic Functions

- For the cubic functions, determine the following:
- $f(x) = x^3 - 5x^2 + 3x + 4$
- $f(x) = -2x^3 + 8x^2 - 5x + 3$
- $f(x) = -3x^3 - 15x^2 - 9x + 27$
- (1) Leading coefficient (2) degree
- (3) domain and range (4) evaluating f(-2)
- (5) zeroes or roots (6) y-intercept
- (7) Symmetry (8) turning points
- (9) maximum values (local and absolute)
- (10) minimum values (local and absolute)
- (11) intervals of increase and intervals of decrease
- (12) end behaviour (+x) and end behaviour (-x)

Conclusions for Cubic Functions

- 1. Describe the general shape of a cubic function
- 2. Describe how the graph of a cubic function with a positive leading coefficient is different than a cubic with a negative leading coefficient
- 3. What does a_0 represent on the graph of a cubic?
- 4. How many real roots do/can cubic functions have?
- 5. How many complex roots do/can cubic functions have?

Conclusions for Cubic Functions

- 6. How many turning points do/can cubic functions have?
- 7. How many intervals of increase do/can cubic functions have?
- 8. How many intervals of decrease do/can cubic functions have?
- 9. Describe the end behaviour ($\pm x$) of a cubic with a (i) positive (ii) negative leading coefficient
- 10. Are cubic functions symmetrical? (You may need to investigate further)

(ii) Quartic Functions

- For the quartic functions, determine the following:
- $f(x)= -2x^4-4x^3+3x^2+6x+9$
- $f(x)= x^4-3x^3+3x^2+8x+5$
- $f(x) = \frac{1}{2}x^4-2x^3+x^2+x+1$

- (1) Leading coefficient (2) degree
- (3) domain and range (4) evaluating f(-2)
- (5) zeroes or roots (6) y-intercept
- (7) Symmetry (8) turning points
- (9) maximum values (local and absolute)
- (10) minimum values (local and absolute)
- (11) intervals of increase and intervals of decrease
- (12) end behaviour (+x) and end behaviour (-x)

Conclusions for Quartic Functions

- 1. Describe the general shape of a quartic function
- 2. Describe how the graph of a quartic function with a positive leading coefficient is different than a quartic with a negative leading coefficient
- 3. What does a_0 represent on the graph of a quartic?
- 4. How many real roots do/can quartic functions have?
- 5. How many complex roots do/can quartic functions have?

Conclusions for Quartic Functions

- 6. How many turning points do/can quartic functions have?
- 7. How many intervals of increase do/can quartic functions have?
- 8. How many intervals of decrease do/can quartic functions have?
- 9. Describe the end behaviour ($\pm x$) of a quartic with a (i) positive (ii) negative leading coefficient
- 10. Are quartic functions symmetrical? (You may need to investigate further)

(iii) Quintic Functions

- For the quintic functions, determine the following:
- $f(x)= x^5+7x^4-3x^3-18x^2-20$
- $f(x)= -\frac{1}{4}x^5+ 2x^4-3x^3+3x^2+8x+5$
- $f(x)=(x^2-1)(x^2-4)(x+3)$

- (1) Leading coefficient (2) degree
- (3) domain and range (4) evaluating f(-2)
- (5) zeroes or roots (6) y-intercept
- (7) Symmetry (8) turning points
- (9) maximum values (local and absolute)
- (10) minimum values (local and absolute)
- (11) intervals of increase and intervals of decrease
- (12) end behaviour (+x) and end behaviour (-x)

Conclusions for Quintic Functions

- 1. Describe the general shape of a quintic function
- 2. Describe how the graph of a quintic function with a positive leading coefficient is different than a quintic with a negative leading coefficient
- 3. What does a_0 represent on the graph of a quintic?
- 4. How many real roots do/can quintic functions have?
- 5. How many complex roots do/can quintic functions have?

Conclusions for Quintic Functions

- 6. How many turning points do/can quintic functions have?
- 7. How many intervals of increase do/can quintic functions have?
- 8. How many intervals of decrease do/can quintic functions have?
- 9. Describe the end behaviour ($\pm x$) of a quintic with a (i) positive (ii) negative leading coefficient
- 10. Are quintic functions symmetrical? (You may need to investigate further)

(D) Examples of Algebraic Work with Polynomial Funtions

- ex 1. Expand & simplify
  $h(x) = (x-1)(x+3)^2(x+2)$.
- ex 2. Where are the zeroes of h(x)?
- ex 3. Predict the end behaviour of h(x).
- ex 4. Predict the shape/appearance of h(x).
- ex 5. Use a table of values to find additional points on h(x) and sketch a graph.
- ex 6. Predict the intervals of increase and decrease for h(x).
- ex 7. Estimate where the turning points of h(x) are. Are the max/min? and local/absolute if domain was [-4,1]

(D) Examples of Algebraic Work with Polynomial Funtions

- ex 8. Sketch a graph of the polynomial function which has a degree of 4, a negative leading coefficient, 3 zeroes and 3 turning points
- ex 9. Equation writing: Determine the equation of a cubic whose roots are -2, 3,4 and f(5) = 28
- ex 10. Prepare a table of differences for $f(x) = -2x^3 + 4x^2 - 3x - 2$. What is the constant difference and when does it occur? Is there a relationship between the equation and the constant difference? Can you predict the constant difference for $g(x) = 4x^4 + x^3 - x^2 + 4x - 5$?

(E) Internet Links

- Polynomial functions from The Math Page
- Polynomial Functions from Calculus Quest
- Polynomial Tutorial from WTAMU
- Polynomial Functions from AnalyzeMath

(E) Homework

- Ex. 7.1 (p. 429) # 13-21 odds, 26-36 evens, 45-57 odds, 62,63
- Ex. 7.2 (p. 438) # 13-23 odds, 29-37 odds