Lesson 15 – Algebra of Quadratics – The Quadratic Formula

Math 2 Honors - Santowski

Fast Five

- (1) If $f(x) = x^2 + kx + 3$, determine the value(s) of k for which the minimum value of the function is an integer. Explain your reasoning
- (2) If $y = -4x^2 + kx - 1$, determine the value(s) of k for which the maximum value of the function is an integer. Explain your reasoning

Lesson Objectives

- Express a quadratic function in standard form and use the quadratic formula to find its zeros
- Determine the number of real solutions for a quadratic equation by using the discriminant
- Find and classify all roots of a quadratic equation

(A) Solving Equations using C/S

- Given the equation $f(x) = ax^2 + bx + c$, determine the zeroes of f(x)
- i.e. Solve $0 = ax^2 + bx + c$ by completing the square

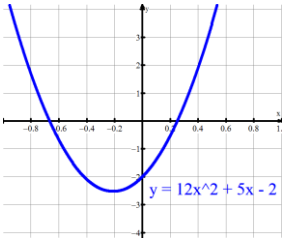
(A) Solving Equations using C/S

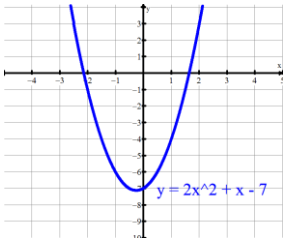
- If you solve $0 = ax^2 + bx + c$ by completing the square, your solution should look familiar:
- $$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$
- Which we know as the quadratic formula
- Now, PROVE that the equation of the axis of symmetry is x = -b/2a

(B) Examples

- Solve $12x^2 + 5x - 2 = 0$ using the Q/F. Then rewrite the equation in factored form and in vertex form
- Determine the roots of $f(x) = 2x^2 + x - 7$ using the Q/F. Then rewrite the equation in factored form and in vertex form
- Given the quadratic function $f(x) = x^2 - 10x - 3$, determine the distance between the roots and the axis of symmetry. What do you notice?
- Determine the distance between the roots and the axis of symmetry of $f(x) = 2x^2 - 5x + 1$

(B) Examples

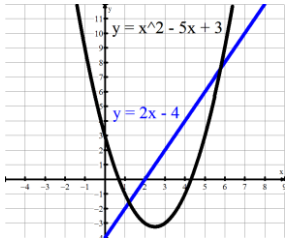




(B) Examples

- Solve the system

$$\begin{cases} y = x^2 - 5x + 3 \\ y = 2x - 4 \end{cases}$$



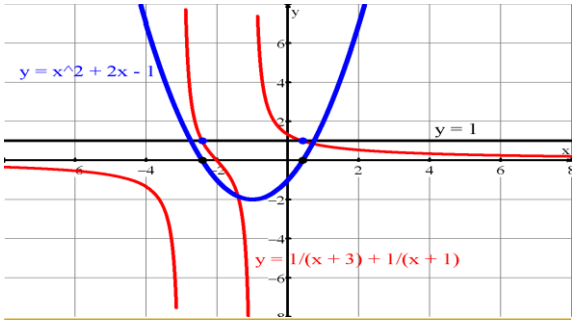
(B) Examples

- Solve the equation and graphically verify the 2 solutions

$$\frac{1}{x+3}+\frac{1}{x+1}=1$$

- Find the roots of $9(x-3)^2 - 16(x+1)^2 = 0$
- Solve $6(x-1)^2 - 5(x-1)(x+2) - 6(x+2)^2 = 0$

(B) Examples

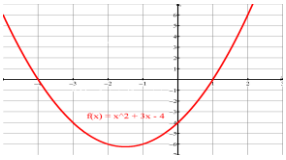


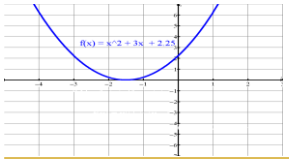
(C) The Discriminant

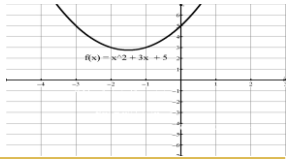
- Within the Q/F, the expression $b^2 - 4ac$ is referred to as the discriminant
- We can use the discriminant to classify the "nature of the roots" ➔ a quadratic function will have either 2 distinct, real roots, one real root, or no real roots ➔ this can be determined by finding the value of the discriminant
- The discriminant will have one of 3 values:
  - $b^2 - 4ac > 0$ ➔ which means ➔
  - $b^2 - 4ac = 0$ ➔ which means ➔
  - $b^2 - 4ac < 0$ ➔ which means ➔

(C) The Discriminant

- Determine the value of the discriminants in:
  - (a) $f(x) = x^2 + 3x - 4$
  - (b) $f(x) = x^2 + 3x + 2.25$
  - (c) $f(x) = x^2 + 3x + 5$







(D) Examples

- Based on the discriminant, indicate how many and what type of solutions there would be given the following equations:
- (a) $3x^2 + x + 10 = 0$
- (b) $x^2 – 8x = -16$
- (c) $3x^2 = -7x - 2$
- Verify your results using (i) an alternate algebraic method and (ii) graphically

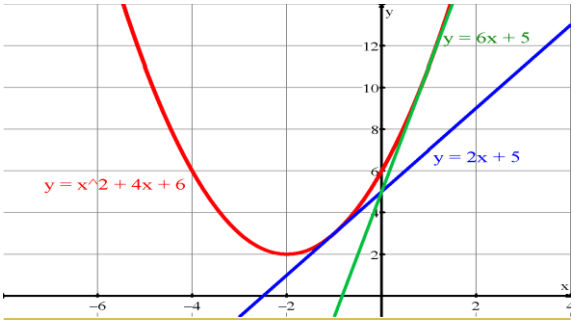
(D) Examples

- Solve the system for *m* such that there exists only one unique solution

$$\begin{cases} y = x^2 + 4x + 6 \\ y = mx + 5 \end{cases}$$

- The line(s) y = mx + 5 are called tangent lines ➔ WHY?
- Now, determine the average rate of change on the parabola (slope of the line segment) between $x_1 = a$ and $x_2 = a + 0.001$ where (a,b) represents the intersection point of the line and the parabola
- Compare this value to *m*.
- What do you notice?

(D) Examples



(D) Examples

- Determine the value of *W* such that $f(x) = Wx^2 + 2x – 5$ has one real root. Verify your solution (i) graphically and (ii) using an alternative algebraic method.
- Determine the value of *b* such that $f(x) = 2x^2 + bx – 8$ has no solutions. Explain the significance of your results.
- Determine the value of *b* such that $f(x) = 2x^2 + bx + 8$ has no solutions.
- Determine the value of c such that $f(x) = f(x) = x^2 + 4x + c$ has 2 distinct real roots.
- Determine the value of c such that $f(x) = f(x) = x^2 + 4x + c$ has 2 distinct real rational roots.

(E) Examples – Equation Writing and Forms of Quadratic Equations

- (1) Write the equation of the parabola that has zeroes of –3 and 2 and passes through the point (4,5).
- (2) Write the equation of the parabola that has a vertex at (4, −3) and passes through (2, −15).
- (3) Write the equation of the parabola that has a y – intercept of –2 and passes through the points (1, 0) and (−2,12).

(F) Homework

- p. 311 # 11-21 odds, 39-47 odds, 48-58