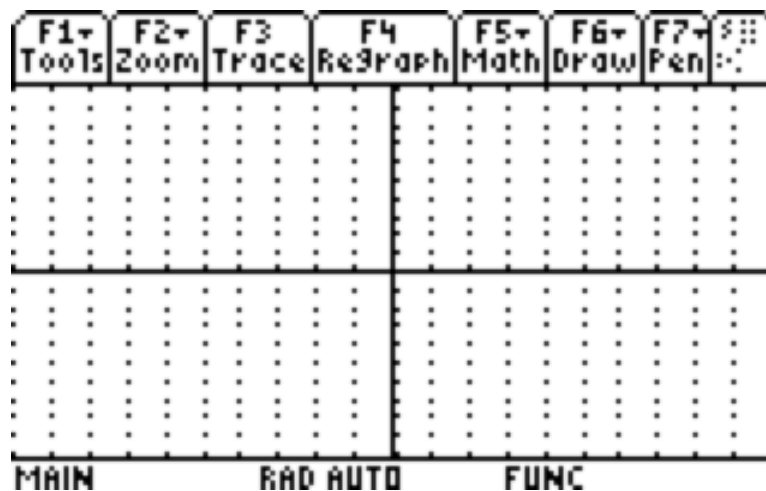


Algebra II: §9.3 Graphs of Rational Functions

Objective: Students will explore the nature of roots in the numerator and denominator of rational functions graphically, algebraically and numerically.

1. Define the function $f(x) = \frac{x^2 - 3x + 2}{x^2 - 4x + 3}$ on your TI-89.
2. Factor the function fully but DO NOT REDUCE (just for the exploration).
3. Sketch the function in a standard zoom window and copy the function in the grid:



4. Identify the coordinates of the x - intercept(s) and the equation(s) of the vertical asymptotes.
 - a) graphically
 - b) algebraically
5. Are there any surprises with respect to the x values you found algebraically compared with the values that you spotted graphically?
6. In the GRAPH window,
 - a) use the F3 Trace key to approach $x = 1$ from the right and left.
 - b) use the F5 Value feature to determine the y - value when $x = 1$.
 - c) What do you notice?

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7. Make a table of values, starting from $x = 0$ and increasing by 1. Identify the x -values for which the function
 - a) has an x -intercept
 - b) has a vertical asymptote
 - c) produces other "unusual" y -values. Explain this x -value. How can this x -value be identified from the factored function? Is the y -value produced by this x -value really undefined? Why or why not (and what name do we give it)?

8. Go to the HOME screen and determine the limit of $f(x)$ as x approaches 1. (Write the proper limit notation here.) Does this value make sense from the graph? Explain.

9. Holes are points that the graph seems to go through, but upon closer inspection are "missing" from the graph. Is $\left(1, \frac{1}{2}\right)$ a point or a hole of $f(x)$? Explain.

10. Now, reduce the function and state the restrictions. Evaluate the REDUCED function for $x = 1$. What do you notice?

11. How can you predict from the original (unreduced) function if the graph will have a hole?

12. If the graph of a rational function does have a hole, how can you determine its
 - a) x -coordinate?
 - b) y -coordinate (no calculator)?

Example: Given the function $g(x) = \frac{x^2 - 3x - 4}{x^2 + x}$,

- a) factor fully, reduce and state any restrictions on x .
- b) determine the coordinates of any holes.
- c) determine the coordinates of the x -intercept(s)
- d) determine the equations of the vertical asymptotes (VAs)
- e) determine the coordinates of the y -intercept
- f) determine the equation of the horizontal asymptote (HA)
- g) sketch the graph and state the domain and range.

Example: Write the equation of one rational function that has all of the following information:

- a) x -intercept at $x = 2$
- b) a hole at $x = 3$
- c) VA at $x = -4$
- d) Modify your function so that the HA is located at $y = 2$.
- e) What is the y -intercept of your function?