ISM: _____ IB: __

Date: ____________________ Block: ______ Name: ____________________________

MATH HONORS 2: UNIT 4 QUIZ 1

Score: ______/ 30 marks ➔ ______ %

Time Allowed: 30 Minutes

Powers and Radicals

1. Simplify $\dfrac{m^3\left(2\dfrac{m}{n^2}\right)^{-2}}{m^0 n^3}$, assuming that *m* and *n* are not equal to zero. Write your answer with positive exponents only. [4 marks]

2. Use algebra to solve for *x* in the following equations: [8 marks]

(a) $8^{x-4} = \left(\dfrac{1}{4}\right)^{3x+1}$ (b) $\sqrt{2x+6} + x = 1$

3. Given that $\sqrt{2x} \cdot \sqrt[3]{8x}$ simplifies to $2^{\frac{a}{b}} x^{\frac{c}{d}}$, where $x \geq 0$ and $a, b, c, d \in \mathbb{Z}^+$, find the values of *a*, *b*, *c*, and *d*. [3 marks]

4. Graph $f(x) = 2\sqrt{-x+1}$, and describe the three transformations applied to the parent function $y = \sqrt{x}$ [4 marks]

5. Simplify the following radical expressions: [8 marks]

(a) $\sqrt{20a^4b^3c^2}$ (b) $\dfrac{3\sqrt{3}}{\sqrt{2}} - \dfrac{\sqrt{3}}{2+\sqrt{2}}$

6. Use graphing technology to solve for *x*: $2\sqrt[3]{x} > x^2 - 1$, rounding your final answer to three significant figures. [3 marks]

MATH HONORS 2: UNIT 4 QUIZ 2

Score: ______/ 30 marks ➔ ______ %

Exponentials and Logarithms

1. Solve the following equations for x: [8 marks]

(a) $27^{2-x} = \left(\dfrac{1}{9}\right)^{2x+5}$ (b) $\log_5 x + \log_5(x-2) = \log_5 8$

2. The *x*-coordinate of the intersection of the exponential functions $f(x) = 5^x$ and $g(x) = 2^{x+3}$ can be expressed in the form $x = \dfrac{\log a}{\log b - \log c}$, where a, b, c are real numbers. Find a, b, and c. [6 marks]

3. Given that $X = \log_a p$, $Y = \log_a q$, and $Z = \log_a r$, express $\log_a\left(\dfrac{p^2 r}{q^3}\right)$ in terms of X, Y, and Z. [4 marks]

4. Sketch the graph of $h(x) = -2^x - 2$ on the axes below, clearly labeling any intercepts with coordinates and any asymptotes with equations. [4 marks]

5. A scientist discovers that he has created A_0 grams of a new radioactive substance [3 marks] called Lacsonium, but notes that it is gradually decaying at a rate of 2% per year according to the equation

$A(t) = A_0(1+r)^t$, where $A(t)$ represents the mass of the Lacsonium after t years.
How many years will it take for the Lacsonium to decay to half of its original mass?

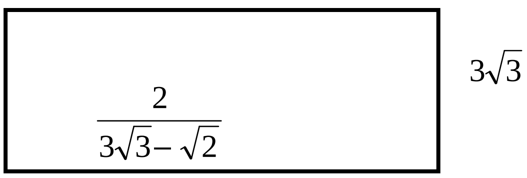
6. On January 1st 2010, Mrs. Brown deposits €1420 in a savings bond that has an annual [5 marks] interest rate of 9% and is **compounded monthly** on the 1st day of each subsequent month. The formula for compounding interest is $A(n) = A_0(1+r)^n$. (HINT: Recall what r and n represent in this context). She wants to take a vacation in Jamaica, but cannot book her ticket until the value of her savings bond exceeds €2150. In what month (and year) can Mrs. Brown book her ticket to Jamaica?

Unit 4 TEST

1. Simplify the expression $\frac{x^3y^{-2}}{x(xy)^4}$. All exponents should be positive. ______________________ (2M)
2. Write $x^{0.8}$ in radical form: ______________________ (1M)
3. Expand $\log\left(\frac{x^3}{\sqrt{z}}\right)^2$. ______________________ (2M)
4. Solve $4^x = \frac{1}{8}$. ______________________ (2M)
5. The equation of the inverse of $h(x) = (x+2)^2 - 3$ is: ______________________ (2M)
6. True or false: $(\ln 3)^2 = \ln 9 = 2\ln 3$. Explain your reasoning. ______________________ (2M)
7. State the exact solution to the equation $2^x = 5$. ______________________ (1M)
8. Determine the value of a, such that $g^{-1}(x) = \log_a(-x)$ and g(-3) = -8.
   ______________________ (1M
9. Add $\sqrt{27} + \sqrt{75} + \sqrt{3}$. ______________________ (2M)
10. Simplify the radical $\sqrt{32a^4b^2c^{-3}}$. ______________________ (2M)
11. The student population of WonFine School is modeled by the equation $P(t) = 1950(0.995)^t$ where *t* is time in years since 1980.
    a. Is the student population growing or declining? ______________________ (1M)
    b. At what rate? (express as a percentage) ______________________ (1M)
12. Graph the function $f(x) = -\ln(x-3)$ on the grid provided. **(3M)**

PART B - Show complete solutions to earn full marks for Questions 13-15

13. Determine the area of the following rectangle: Express all values to correctly simplified radicals: **(3M)**



14. This question deals with 2 functions, $f(x) = \sqrt{x+2}$ and $g(x) = 3 - \frac{1}{2}x$.

    a. Determine the domain in which you predict the solution to the equation $\sqrt{x+2} = 3 - \frac{1}{2}x$ exists. Explain/show your reasoning in determining the domain. **(2M)**

    b. Solve $\sqrt{x+2} = 3 - \frac{1}{2}x$. **(4M)**

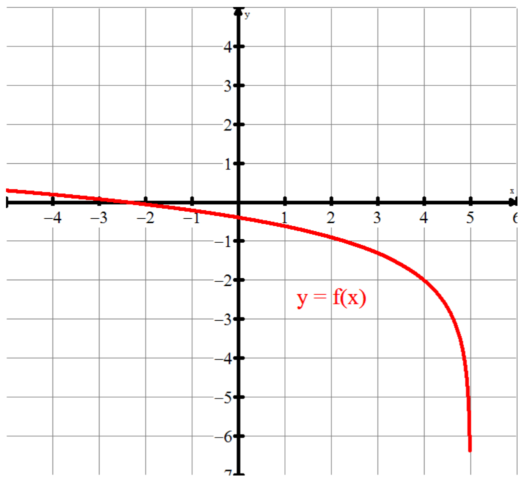
    c. Now solve the inequality $\sqrt{x+2} \le 3 - \frac{1}{2}x$. A grid has been provided for your convenience. You are NOT obligated to use the grid. **(2M)**

15. Solve $\log_2(\log_4(\log_3 x)) = 0$ for x. **(3M)**

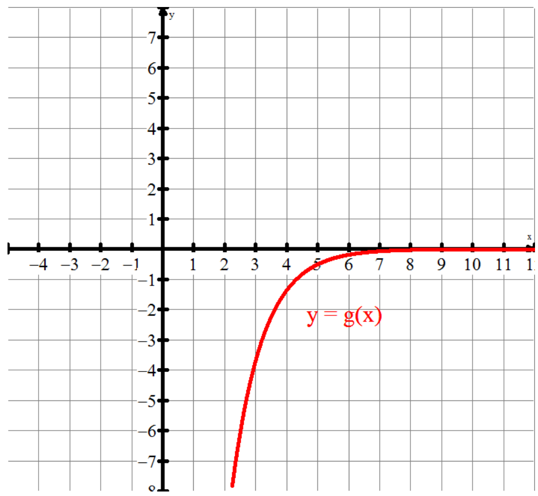
16. Given the following graph and equations, match the appropriate equation to the corresponding graph. Make sure your selection is OBVIOUS i.e. write your chosen equation on the graphs. **(4M)**

$y(x) = -\frac{1}{2}e^{5-x}$	$y(x) = -2\sqrt{5-x}$	$y(x) = -\frac{1}{2}e^{5+x}$	$y(x) = -2+\ln(5+x)$
$y(x) = -2+4^{1-x}$	$y(x) = -2+\left(\frac{1}{4}\right)^{1-x}$	$y(x) = -2+\ln(5-x)$	$y(x) = -2\sqrt{-5-x}$

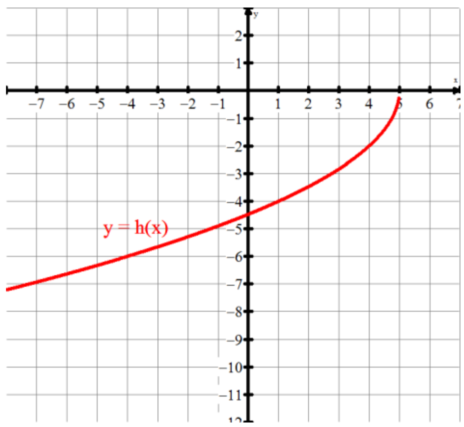
GRAPH A



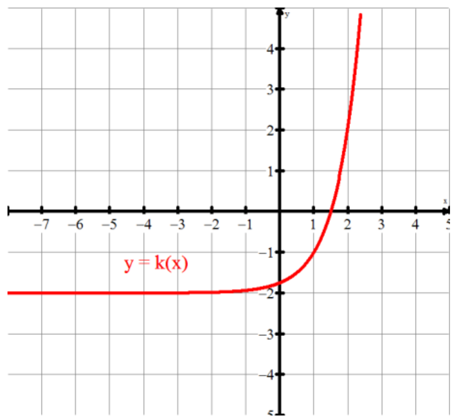
GRAPH B



GRAPH C



GRAPH D



PART C - CALCULATOR ACTIVE – Provide complete solutions, including all key relevant equations & algebraic steps and sketches of relevant graphs and results from the calculator

17. In September 1998 the population of the country Golly Gee in millions was modeled by $P(t) = 16.6e^{0.019t}$. At the same time, the population of Gosh Darn in millions was modeled by $C(t) = 12 + 14.6e^{0.0124t}$. In both formulas, *t* is the year and *t* = 0 corresponds to January 1998. A grid has been provided for you, should you require it. **(6M)**

a. What is the initial population of each town? **(1M)**
b. Which town population grows faster? How do you know? **(2M)**
c. When to the nearest year will the two countries have equal populations? **(2M)**
d. What assumption are you making in your solution to (c)? **(1M)**

18. Dry cleaners use a cleaning fluid that is purified by evaporation and condensation after each cleaning cycle. Every time it is purified, 2% of the fluid is lost.
    a. An equipment manufacturer claims that after 20 cycles, about two-thirds of the fluid remains. Verify or reject this claim. **(2M)**
    b. If the fluid has to be "topped up" when half the original amount remains, after how many cycles should the fluid be topped up? **(2M)**
    c. A manufacturer has developed a new process such that two thirds of the cleaning fluid remains after 40 cycles. What percentage of fluid is lost after each cycle? **(2M)**

19. Solve $3\left(2^{2x-1}\right)=4^{-x}$. Provide an exact solution, showing the key algebra steps of your solution. **(5M)**
20. Solve for *x* and verify $\log_2\left(x^2-6x\right)=3+\log_2(1-x)$. Provide an algebraic solution to potentially earn full marks. **(5M)**

MATH HONORS 2: UNIT 5 QUIZ 1

Score: ______ / 30 marks ➔ ______ %

Triangle and Circle Trigonometry

1. An angle θ is co-terminal to $-640°$. [2 marks]

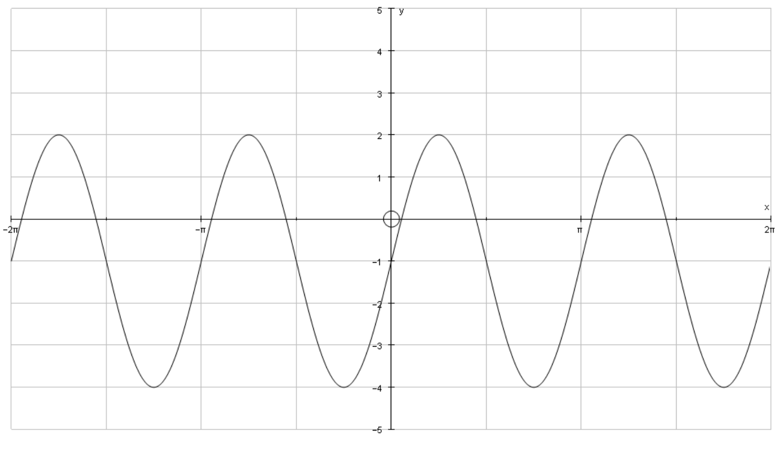
(a) Given that $0° \le \theta \le 360°$, find θ. (b) Convert θ to radians (in terms of π).

2. Evaluate: (a) $\tan(330°)$ (b) $\csc\left(\frac{3\pi}{4}\right)$

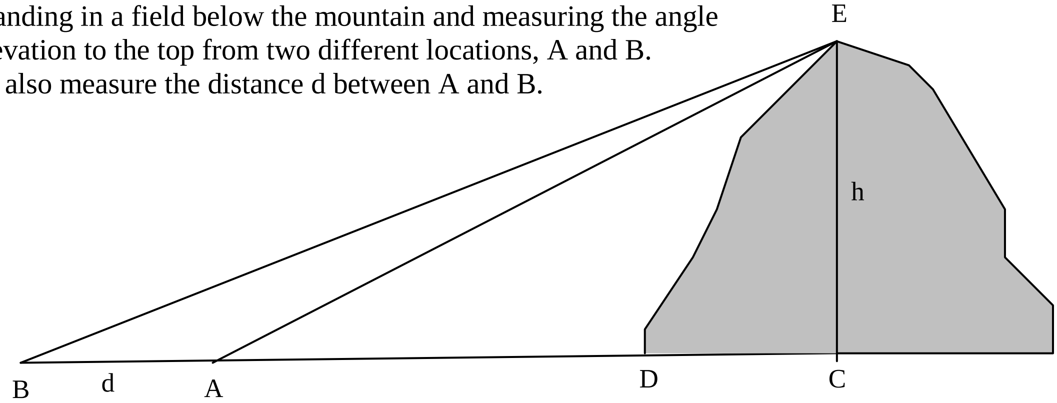
3. Solve $\cos x=\frac{1}{2}$ for all solutions of x, given that $-\pi \le x \le 2\pi$ [3 marks]

4. Given that $\sin\theta=\frac{2}{3}$ and θ is in the second quadrant, find $\sec\theta$. [3 marks]

5. Use the graph of the function $f(x)=a\sin(bx)+d$ to find the values of a, b, and d. [3 marks]



6. A group of surveyors measure the vertical height of a mountain by standing in a field below the mountain and measuring the angle of elevation to the top from two different locations, A and B. They also measure the distance d between A and B. [6 marks]



If angle EBC is 27°, angle EAC is 34°, and d = 862 m, find the vertical height h of the mountain, rounding your answer to the nearest meter.

7. Find the possible values for $m\angle B$ in triangle ABC, given $m\angle A = 33°$, a = 10 cm, and b = 13 cm. Round your answers to the nearest degree.

8. A farmer has a triangular field that is bordered by one stone wall and two fences. The fences meet at an angle of $71°$, and have lengths of 49 meters and 68 meters. [5 marks]

(a) Find the area of the farmer's field.

(b) Find the length of the stone wall.

MATH HONORS 2: UNIT 5 QUIZ 2

Score: ______ / 30 marks ➔ ______ %

Trigonometric Graphs, Identities, and Equations

1. The exact value of $\cos 15° + \sin 105°$ can be expressed in the form $\dfrac{\sqrt{a}+\sqrt{b}}{2}$, where a and b are positive integers. Find a and b.

2. Prove the following trigonometric identities: [7 marks]

(a) $\dfrac{1}{2}(\tan x + \cot x) = \dfrac{1}{\sin(2x)}$ (b) $\dfrac{\cot\theta}{\csc\theta + 1} = \dfrac{\csc\theta - 1}{\cot\theta}$

3. Given that $\theta = \arctan\left(\dfrac{-3}{4}\right)$, find the exact value of $\cos 2\theta$. [4 marks]

4. Solve the following equations for all solutions of θ in the given domains: [8 marks]

(a) $2\sin^2\theta - \cos\theta = 1,\ -\pi \le x \le \pi$ (b) $\tan(3x) + 2 = 1,\ 0° \le x \le 180°$

5. Explain why the equation $\sec\theta = \dfrac{\pi}{4}$ has no solutions. [2 marks]

6. Graph the function $y = \arccos(x-1)$, providing scales on the axes below and labeling any intercepts with coordinates. [4 marks]

UNIT 5 TEST - TRIG

PART A – The following questions are CALCULATOR INACTIVE. Provide clear, concise solutions that show the key algebraic steps in your solutions:

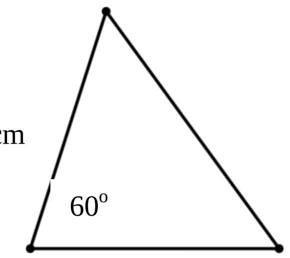
1. Evaluate $\cot\left(\cos^{-1}\left(\frac{5}{13}\right)\right)$. **(2M)**

2. Write a trigonometric equation that produces the solutions $\theta = \frac{5\pi}{6}, \frac{11\pi}{6}$ on the domain $\theta \in [0, 2\pi]$.

3. The solution to $\cot(x) = -1$ on an infinitely defined domain is: **(2M)**

4. In which quadrant does θ lie if $(\sin\theta)(\cos\theta) < 0$ and $\csc\theta < 0$? Justify your answer. **(2M)**

5. Solve for the unknown side in the diagram below: **(2M)**



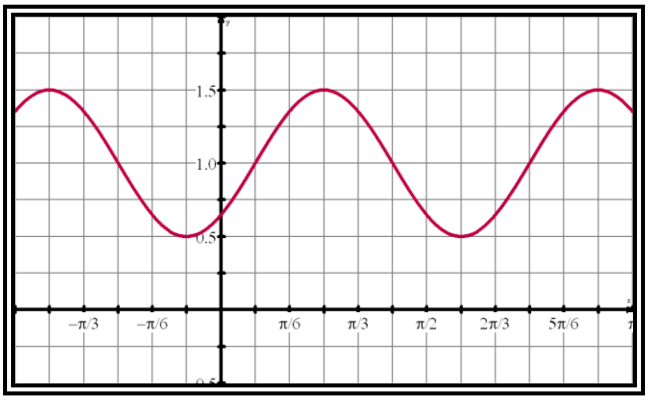
6. In this question, you will work with the function $f(\theta) = 2\sec\frac{1}{4}\left(\theta - \frac{3\pi}{2}\right)$ and your knowledge of transformations. A grid has been provided for your use (if you wish). **(6M)**
   a. Write the equation of a vertical asymptote of $f(\theta) = \sec(\theta)$.
   b. The equations of any two (2) vertical asymptotes of $f(\theta) = 2\sec\frac{1}{4}\left(\theta - \frac{3\pi}{2}\right)$
   c. The co-ordinates of one maximum point of $f(\theta) = \sec(\theta)$.
   d. The co-ordinates of one maximum point of $f(\theta) = 2\sec\frac{1}{4}\left(\theta - \frac{3\pi}{2}\right)$.
   e. The range of $f(\theta) = 2\sec\frac{1}{4}\left(\theta - \frac{3\pi}{2}\right)$.

7. Prove the identity $\frac{\sin(2x)}{1+\cos(2x)} = \tan(x)$ and then state and EXPLAIN the domain of validity for this identity. **(6M)**

8. Determine the exact ratio for $\sec\left(\frac{5\pi}{12}\right)$. (HINT: addition/subtraction identities??) **(4M)**

9. You are given a graph of a sinusoidal function $y = f(\theta)$. **(5M)**

   a. Determine an appropriate sinusoidal equation for the function.
   b. Graph the inverse **function**, $y = f^{-1}(\theta)$ on the second graph. Clearly label/indicate the key points and place an appropriate scale on the axis (axis NOT included in second graph).



PART B – The following questions are CALCULATOR ACTIVE. Provide clear, concise solutions that show the key algebraic steps.

10. Solve the equation $4\sin(\theta)+3=0$ for $\theta \in [-180°,360°]$. Round answer(s) to the nearest degree. **(2M)**

11. The angle $\theta = -\dfrac{47\pi}{13}$ is given. **(3M)**

    a. Convert the angle to degrees, correct to the nearest degree.
    b. Sketch θ in standard position on grid (show the rotation(s)):
    c. State the measure (in radians) of an angle that is coterminal with θ.

12. How many cycles of the sinusoid $y=\cos(kx),\ k\in R$ are there from $x\in[-2\pi,2\pi]$? **(2M)**

13. Prove the identity $\tan(x)\sin(x)=\sec(x)-\cos(x)$. **(3M)**

14. In this question, you will determine the co-ordinates of the intersection points of the graphs of $f(\theta)=\sin(2\theta)$ and $g(\theta)=\cot(\theta)$ on the interval $[0,2\pi]$.

    a. Illustrate the graphical solution from your calculator
    b. Provide an ALGEBRAIC solution to the question. **(5M)**

15. A Ferris wheel with a radius of 7 meters makes one complete revolution every 16 seconds. The bottom of the Ferris wheel is 2 meters above the ground, where the riders get on and off the ride. **(8M)**

    a. On the grid provided, draw a graph showing how a rider's height varies with time for 2 revolutions of the Ferris wheel, given that the rider enters the Ferris wheel at the bottom of the wheel.
    b. Determine an equation for the graph.
    c. At what height above the ground is a rider after 27 seconds? Round final answer to the nearest 10^{th} of a meter. Explain/show algebraically HOW you determined your answer.
    d. At what time(s) in the 2 revolutions is the rider 12m above the ground? Round final answer to the nearest 10^{th} of a second. Explain/show algebraically HOW you determined your answer.