

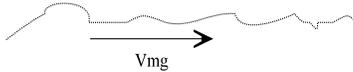
Relative Velocity in 2 - Dimensions

There are only two options for crossing a river (or flying a plane in a wind) that deal with right triangles. You can either head straight across the current and get pushed downstream, or you can angle yourself into the current so as to arrive straight across from where you started. The following example will look at both types of motion.

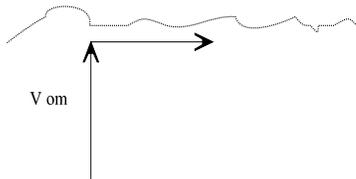
- In still water, Mr. Furey can swim at 5.0 km/hr. If a river is flowing east at 3.0 km/hr.
  - If Mr. Furey **heads straight across**, what will be his velocity relative to the ground?
  - At what angle** (heading) must Mr. Furey point in order to arrive straight across the river?

Head straight across, get pushed downstream

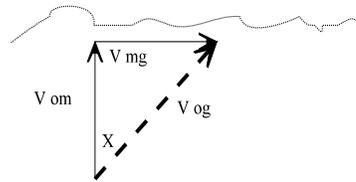
- Draw diagram of the river with the current (it is easier to put the current on the opposite side of the bank to where you are)



- Draw the vector  $V_{om}$  (your velocity in still water) as a straight line across, be sure you connect tip to tail.   
 \* notice the hypotenuse is our unknown



- Draw your resultant vector  $V_{og}$ , it should be *at an angle*.



- Calculate the magnitude (size) of  $v_{og}$  using Pythagoras theorem.

$$\begin{aligned} v_{og}^2 &= v_{om}^2 + v_{mg}^2 \\ &= 5^2 + 3^2 \\ &= 25 + 9 \\ &= 34 \end{aligned}$$

- Calculate the angle using trig.

$$\begin{aligned} \sin x &= \frac{\text{opp}}{\text{hyp}} \\ x &= \sin^{-1}(3/5.8) \end{aligned}$$

$$v_{og} = 5.8 \text{ km/hr}$$

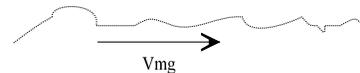
$$x = 31^\circ$$

- Write your resultant vector

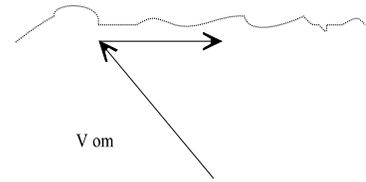
$$v_{og} = 5.8 \text{ km/hr [N31E]}$$

Head into the current, end up straight across

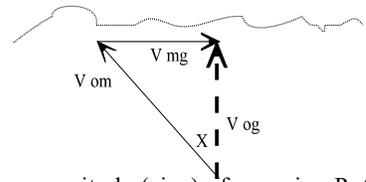
- Draw diagram of the river with the current (it is easier to put the current on the opposite side of the bank to where you are)



- Draw the vector  $V_{om}$  (your velocity in still water) on an angle, be sure you connect tip to tail.   
 \* notice the hypotenuse is **not** our unknown



- Draw your resultant vector  $V_{og}$ , it should be *straight across*.



- Calculate the magnitude (size) of  $v_{og}$  using Pythagoras

$$\begin{aligned} v_{om}^2 &= v_{og}^2 + v_{mg}^2 \\ 5^2 &= v_{og}^2 + 3^2 \\ 25 &= v_{og}^2 + 9 \\ 16 &= v_{og}^2 \end{aligned}$$

$$v_{og} = 4.0 \text{ km/hr}$$

- Calculate the angle using trig.

$$\begin{aligned} \sin x &= \frac{\text{opp}}{\text{hyp}} \\ x &= \sin^{-1}(3/5) \end{aligned}$$

$$x = 37^\circ$$

- Write your resultant vector

$$v_{og} = 4.0 \text{ km/hr [N37W]}$$

Practice: \_\_\_\_\_

1. Julia can swim at 3.5 km/hr in still water. She attempts to head straight north across a river with a current flowing at 1.2 km/hr [W]. What is her resultant velocity  $v_{og}$ ? (3.7 km/hr [N19°W])
2. In problem #1, at what angle should Julia aim in order to land directly across the river from her start point? (must aim at [N20°E])
3. An aircraft has an airspeed of 230 km/hr [N]. What is the plane's speed relative to the ground if it:
  - a) Flies with a tailwind of 50 km/hr? (280 km/hr [N])
  - b) Flies into a headwind of 50 km/hr? (180 km/hr [N])
  - c) Flies due [N] while the wind blows at 50 km/hr [E]? (235 km/hr [N12°E])
4. An aircraft leaves Albany for New York, which is due south. If the aircraft can fly at 450 km/hr, and the wind is blowing to the west at 40 km/hr, what must be the aircraft's heading in order to fly due south? ([S5°E])  
(Heading is the angle of the planes velocity relative to the ground)  
(Ground speed is the magnitude of the plane's velocity relative to the ground)
5. A navy vessel is patrolling the Straights of Hormuz for oil smugglers. The ship can travel at 30 km/hr in still water. If the current is 3 km/hr [W], what must be the ship's heading to maintain a course due [N]? (head [N5.7°E])
6. A swimmer jumps into a river and swims straight for the other side at 1.5 km/hr relative to the water. There is a current in the river of 2.0 km/hr [W]. What is the swimmer's velocity relative to the shore? (2.5 km/hr [N53°W])
7. A conductor in a train travelling at 12.0 km/hr [N] walks across the aisle at 5.0 km/hr relative to the train. What is his velocity relative to the ground? (13.0 km/hr [N23°E])
8. A mouse is crawling inside a box which is being pushed across the deck of a moving boat. The mouse is travelling at 0.5 m/s [E] relative to the box. The box is moving at 1.2 m/s [E] relative to the boat. The boat is moving at 4.0 m/s [N] relative to the water. What is the velocity of the mouse relative to the water?  
\*Hint...simply add the collinear velocities first, then use pythagoras on the combined East velocity and the north velocity. (4.3 m/s [E67°N])
9. A plane has a velocity of 300 km/hr relative to the air. If the pilot points the plane straight north, when there is a wind of 80 km/hr blowing towards the west, what will the resultant velocity of the plane be?(310 km/hr [N15°W])
10. A blimp pilot wants to travel north . The blimp can move at 26 km/hr in still air. There is a wind of 10 km/hr east.
  - a) What is the heading (which way should he point the blimp) ([N23°W])
  - b) How fast will the blimp travel relative to the ground? (24 km/hr)
11. A plane wants to travel east. The plane has a velocity of 500 km/hr relative to the air. A wind is blowing at 50.0 km/hr [N]. Calculate
  - a) the proper heading ([E5.7°S])
  - b) the magnitude of the plane's velocity relative to the ground (497 km/hr)